NETWORK THEORY

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CHAPTER 1

BASIC CIRCUIT

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The interconnection of various electric elements in a prescribed manner comprises an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc.Analysis of electric refers to computation required to determine the unknown quantities such as voltage current and power conium one or more in the circuit element. To contribute to the solution in engineering problems the must gene the basic understanding of electric circuit laws and circuit analysis. The other system, like mechanical, hydraulic, thermal magnetic and power system and easy to examine and modal by a circuit. The learn how to examine the models of these system first one needs to learn the techniques of circuit analysis. We shall discuss about thr briefly some of the basic circuit elements and will help us to grow the backdrop of subject

Basic Elements Introductory Concepts

Electrical Network: A blend of various electric elements (Resistor, Inductor, Voltage source, Current source, Capactor) join in any manner that is called an electrical network. Us classify circuit element in two categories,

The first categories is **– Active Element**

The second categories is – **Passive Element**

Active Element: The element that supply energy to the circuit is called active element. The example of active element take in voltage and current sources, generators, and electronic device that power supply need .A transistor is a circuit element that is now active, meaning that it can make louder power signal. On the order hand transformer is not an active element because it does not louder because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element[1]–[3].

Passive Element: The element which be given energy and then either converts it into heat or stored it in an electric or captivating field is called passive element.

Bilateral Element: Transmission of current in both management in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.

Unilateral Element: Conduction of current in one management is termed as one-sided (example: Diode, Transistor) element.

Meaning of Response: An appeal of input signal to the system will give rise to an output signal, the way of behaving of output signal with time is known as the reciprocation of the system.

Potential Energy Difference: The voltage or potential energy distinctness between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

Ohm's Law: Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$I = V / R$$

$$I = \frac{V}{R}$$

where I is the current through the resistance in units of amperes, V is the possible difference measured across the resistance in units of volts, and R is the resistance of the conductor in units of ohms. More exactly, Ohm's law states that the R in this relation is constant, independent of the current.

Kirchoff's law

Kirchoff's First Law

The Current Law, (KCL) - The total current or charge incoming a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node. In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero (Figure 1.1).

$$I(exiting)+I(entering)=0$$

This idea by Kirchoff is known as the Conservation of Charge.

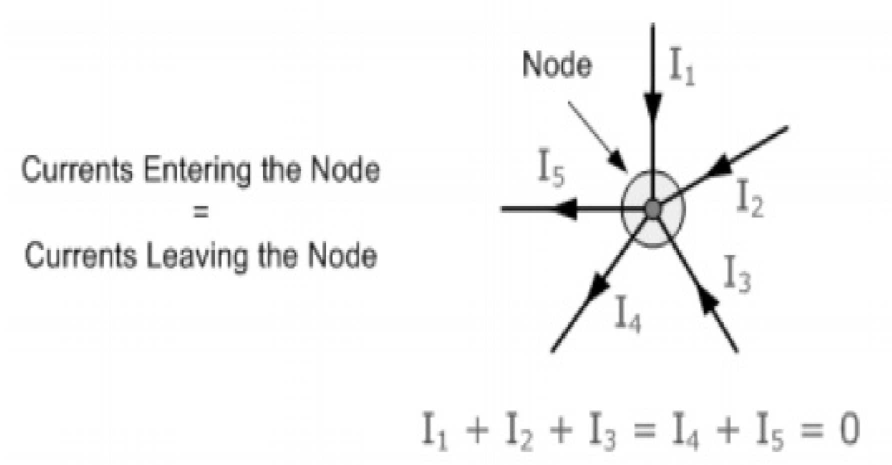

Figure 1.1 The Current Law

Here, the 3 currents ingoing the node, I1, I2, I3 are all positive in value and the 2 currents leaving the node, I4 and I5 are negative in value[4]–[6].

Then this means we can also rewrite the equation as; I1 + I2 + I3 - I4 - I5 = 0

Kirchoff's Second Law - The Voltage Law, (KVL)-In any closed circle network, the total power around the loop is equal to the amount of all the voltage drops within the same loop″ which is also equal to zero. In other words the numerical sum of all voltages within the circle must be equal to zero. This idea by Kirchoff is known as the Saving of Energy**.** Starting at any idea in the loop remain in the similar direction seeing the direction of all the voltage drips, also positive or negative, and frequent back to the same starting point. It is important to keep the same course either clockwise or anti-clockwise or the final voltage sum will not be equal to zero (Figure 1.2).

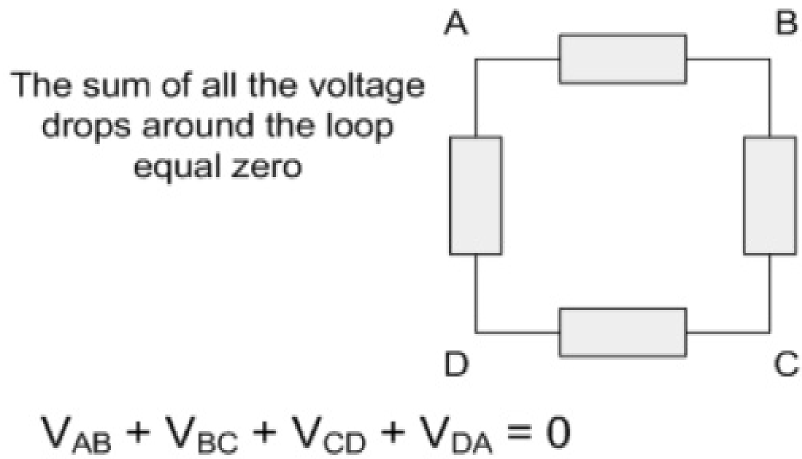

Figure 1.2: Kirchoff's Second Law

DC Circuits: A DC circuit (Direct current circuit) is an electrical circuit that contains of any condition of endless voltage sources, endless current sources, and devices. In this case, the circuit voltages and currents are constant, i.e., free of time. More technically, a DC circuit has no memory. That is, a precise circuit voltage or current does not be liable on the past value of any circuit voltage or current. This suggests that the system of calculations that denote a DC circuit do not include integrals or by products.

If a capacitor and inductor is added to a DC circuit, the ensuing circuit is not, sternly speaking, a DC circuit. Though, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC fixed state. More exactly, such a circuit is represented by a system of differential balances. The solutions to these self-controls usually contain in a time variable or brief part as fine as endless or stable state part. It is this fixed state part that is the DC result. Here are some circuits that do not need a DC response. Two simple examples are a constant current source connected to a capacitor and a endless voltage basis connected to an inductor.

In electronics, it is joint to refer to a circuit that is powered by a DC voltage source such as a batery or the output of a DC power supply as a DC circuit even still whatever is meant is that the circuit is DC powered[7]–[10].

AC Circuits:

Fundamentals of AC: An alternating current (AC) is an electrical current, wherever the magnitude of the current varies in a cyclic form, as different to direct current, where the split of the current stays constant.

The normal waveform of an AC circuit is normally that of a sine wave, by way of these results in the well-organized transmission of energy. But in certain requests different waveforms are used, such as triangular or square waves. Used generically, AC refers to the system in which electricity is sent to businesses and residences. However, audio and radio signals carried on electrical wire are also cases of alternating current. In these applications, an chief goal is often the recovery of gen fixed (or modulated) onto the AC signal.

Difference Between AC and DC:

Current that flows constant in one way is called direct current broken current (A.C) is the current that currents in single way for a short-term time before converses and flows in different way for a parallel time. The basis for alternating current is called ac generator or alternator.

Cycle: One whole set of positive and negative values of a broken quantity is called cycle.

Frequency: The amount of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

Amplitude or Peak value: The punishing positive or negative value of an alternating number is called fullness or peak value.

Average value: This is the average of instant values of an alternating quantity over one complete cycle of the wave.

Time period: The time taken to wide-ranging one complete cycle.

Average value derivation: Let I = the express value of current and I = Im sin Where, Im is the maximum value.

Resistors in series and parallel circuits:

Series circuits: Figure shows three resistors R1, R2 and R3 linked end to end, i.e. in series, with a batery basis of V volts. Since the circuit is closed a current I will flow and the p.d. across each resistor may be unwavering from the voltmeter readings V1, V2 and V3.

In a series circuit

The current I is the same in all parts of the circuit and hence the same analysis is found on each of the two ammeters shown, and the sum of the voltages V1, V2 and V3 is equal to the total applied voltage, V, i.e.

$V = V1 + V2 + V3$

From Ohm's law:

V1 = IR1, V2 = IR2, V3 = IR3 and V = IR where R is the total circuit resistance. Since V = V1 + V2 + V3.

Then IR =IR1 + IR2 + IR3 Dividing throughout by I gives R = R1 + R2 + R3

Mash Analysis: This is another planned method toward solving the circuit and is based on computing web currents. A similar method to the node complaint is used. A set of designs is formed are the equations are solving for odd values. As many controls are needed as odd mesh currents be.

Step 1: Classify the mesh currents

Step 2: Switch which mesh currents are known

Step 2: Write equation for each mesh by KVL and that comprises the mesh currents.

Step 1: The mesh undercurrents are as shown in the diagram on the next page

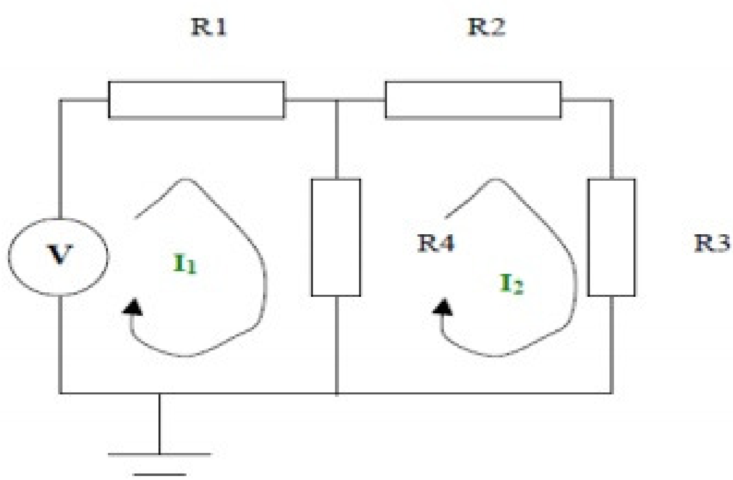

Figure 1.3: Mash Analysis

Step 3:

KVL can be useful to the left hand side loop. This ranks the voltages about the loop sum to zero.

When writing down the voltages across each device calculations are the mesh currents.

I1R1 + (I1 - I2) R4 - V = 0

KVL can be applied to the right hand side loop. This ranks the voltages everywhere the loop sum to

Zero. When script down the voltages across ear the balances are the mesh currents.

I2R2 + I2R3 + (I2 - I1) R4 = 0

Nodal analysis:

Nodal analysis includes looking at a circuit and causal all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop among that node and a position node (usually ground). Once the node currents are known any of the Currents smooth in the circuit can be strong-minded. The node way offers a ready way of achieving this.

Approach:

Firstly all the nodes in the circuited are counted and identified. Next nodes at which the power is already known are recorded. A set of calculations based on the node voltages are molded and these equations are solved for unknown numbers. The set of equations are molded using KCL at each node. The set of real-time controls that is produced is then split. Division currents tin then be start once the node voltages are branded. This can be compact to a runs of steps:

Step 1: Identify the nodes

Step 2: Choose a reference node

Step 3: Identify which node voltages are known if any Step 4: Identify the BRANCH currents

Step 5: Use KCL to write an equation for each strange node voltage

Step 6: Solve the balances

This is best clarified with an example. Finding all currents and voltages in the following circuit using the node way. (In this particular case it can be solved in other ways as well)

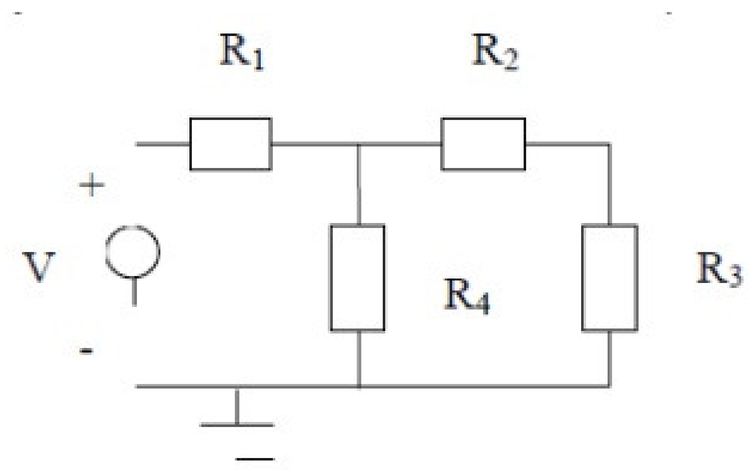

Figure 1.4: Nodal analysis

Step 1: There are four nodes in the circuit. A, B, C and D

Step 2: Ground, node D is the position node.

Step 3: Node voltage B and C are unknown. Voltage at A is V and at D is 0

Step 4: The currents are as shown. There are 3 different currents

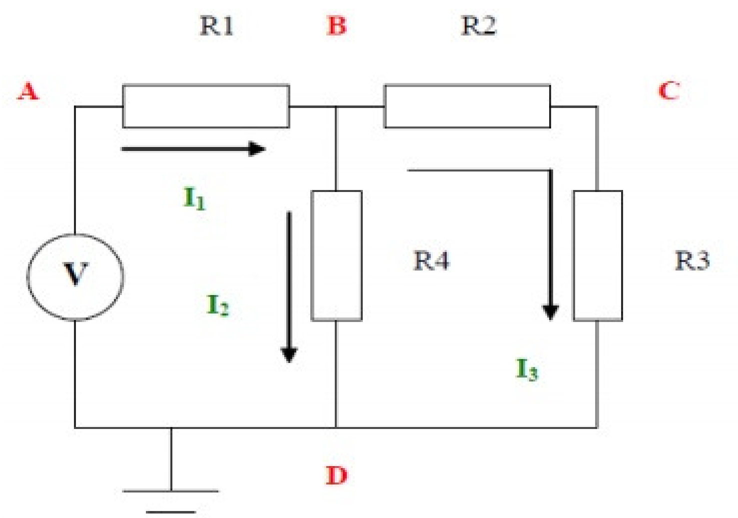

Figure 1.5: Three different currents

Step 5:

I need to create two comparisons so I apply KCL at node B and node C

The statement of KCL for node B is as follows:

$$\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0$$

The statement of KCL for lump C is as follows:

$$\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0$$

Step 6:

We currently have two calculations to solve for the two mysteries VB and VC. Solving the above two comparisons we get:

$$V_C = V\frac{R_3 R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

$$V_B = V\frac{R_4(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

Further calculation

The node voltages are know all branded. From these we can get the outlet currents by a simple application of Ohm's Law:

$$I_1 = (V - V_B) / R_1$$

$$I_2 = (V_B - V_C) / R_2$$

$$I_3 = (V_C) / R_3$$

$$I_4 = (V_B) / R_4$$

In this chapter we will different type circuit element. The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. also study of active element and passive element. A dc circuit (Direct current circuit) is an electrical circuit that contains of any condition of endless voltage sources, endless current sources, and devices.

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CHAPTER 2

NETWORK REDUCTION AND THEOREMS FOR DC AND AC CIRCUITS NETWORK

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The electriic system analysiis, The domiinant rules are Ohm‘s Law and Kirchhoff’s Laws. Whiles these modest rules may be appliieed to study just near any circuts formatiions (even if we have to resort to complex algebra to grip multiiple unknowns), here are some shortcuts deviices of analysiis To Type the calculatiion easiier for the regular human. As wiith any Theorem of geometry or algebra, these system Theorems are deriived from maiin rules. IIn Thiis chapter, II‘m not goiing to delve iinto the offiiciial eviidences of any of These Theorems. IIf you doubt theiir weiight, you can always empirically quiiz them iin setiing actiive sample ciircuiits and calculatiing values by the old concurrent calcuatiion means agaiinst The novel Theorems, To see iif The answers coiinciide.

Network Theorems are also can be called as network reductiion Techniiques. Each and every Theorem became iits iimportance of solviing net. Let us see some important Theorems with DC and AC exciitatiion with full procedures

Tellegen" s theorem:

Dc Excitation: Telligent’s Theorem conditions numeriical sum of all deliivered control must be equal To sum of all usual controls. Giiviing to Telliigent’s Theorem, The summary of instantaneous powers for The n number of branches in an electrical system is zero. Are you confused? Let’s explaiin. Supposiing a number of branches in an electrical system must ii1, ii2, ii3…. in respectiive instantaneous currents done Them. These currents content Kiirchhoff's Current Law. Agaiin, suppose These branches have instantaneous currents across Them are v1, v2, v3, .. v_n independently. IIf These voltages across These basiics content Kiirchhoff Voltage Law Then, [1]–[3]

$$\sum_{k=1}^{n} v_k \cdot i_k = 0$$

V_k is The immediate power crosswiise The k^{Th} branch an iis The iinstantaneous current smooth complete Thiis branch. Tellegen□s Theorem iis appliicable mostly iin general class of Taken networks That consiist of liinear, non-linear, actiive, passive, Tiime variant and Tiime variant elements.

Kiirchhoff’s Laws

These laws are more comprehensiive Than Ohm’s law and are used for solviing electriical networks whiich may not be readiily solved by the later. Kiirchhoff’s laws, two iin number, are maiinly useful

(a) IIn deTermiiniing the equal resiistance of a complex network of conductors and

(b) For calculatiing The currents flowiing iin The variious conductors.

The Two-laws are:

Kirchhoff's Point Law or Current Law (KCL) iit states as follows

IIn any electriical network, The numeriical sum of The currents meetiing at a poiint (or junctiion) iis zero

IIn another way, iit siimply means that the Total current leaviing a junctiion iis equal To the Total current iincomiing that junctiion. IIT iis clearly true because there iis no accumulatiion of charge at The junctiion of The networks.

Consiider The case of a few conductors meetiing at a poiint A as iin Some rods have current's leadiing To poiints A, where as some have current's leadiing away from poiints A. assumiing The incoming current's To be posiitiive and The outgoiing current's negatiive, we have

$$II1 + (-II2) + (-II3) + (+ II4) + (-II5) = 0$$

Or $$II1 + II4 - II2 - II3 - II5 = 0 \text{ or } II1 + II4 = II2 + II3 + II5$$

Electriic Ciircuiits and Network Theorems

There are sure formulae, whiich once appliied To The answers of electriic networks, decliine siimpliify The networks hiimself or reduce Theiir analytiical solutiion very easy. These Theorems can also be appliied to an a.c system, wiith The only diifference That iimpedances replace The osmiic resiistance of D.c. system. Diifferent electriic ciircuiits (accordiing To Theiir propertiies) are defiined below:

Ciircuiit. A ciircuiit iis a closed conductiing path through whiich an electriic current eiither flows or iis iintended flow.

Parameters. The variious elements of an electriic ciircuiit are called iits parameters liike resiistance, iinductance and capaciitance. These parameters may be lumped or diistriibuted.

Liiner Ciircuiit. A liinear ciircuiit iis one whose parameters are constant ii.e. they do not change wiith voltage or current.

Non-linear Circuit. IIT iis That ciircuiit whose parameters change wiith voltage or current.

Biilateral Circuit. A bilateral circuit is one whose propertiies or characteriistiics are the same in either directiion. The usual Transmiissiion liine iis biilateral, because it can be made to perform its functiion equally well iin eiither diirectiion.

Unilateral Ciircuiit. IIT is that circuit whose propertiies or characteriistiics change with The direction of iits operation. A diiode rectiifiier iis a uniilateral ciircuiit, because it cannot perform rectiifiicatiion in both directions. Electriic Network. A combiinatiion of variious electriic elements, connected in any manner whatsoever, is called an electriic networks.

Passiive Network is one whiich contaiins no source of e.m.f in it 9. Actiive Network iis one whiich contaiins one or more Than one source of e.m.f.

Node is a junctiion iin a ciircuiit where two or more circuit elements are connected together.

Branch is that part of a networks whiich liies between two junctions.

Loop. IT is a close path in a circuit in whiich no element or node is encountered more Than once.

Mesh. IT is a loop that contaiins no other loop wiithiin it.

Voltage source to current source transformatiion:

IIf voltage source iis converted To current source, Then The current source II = V/Rse wiith parallel resiistance equal To Rse (Figure 2.1).

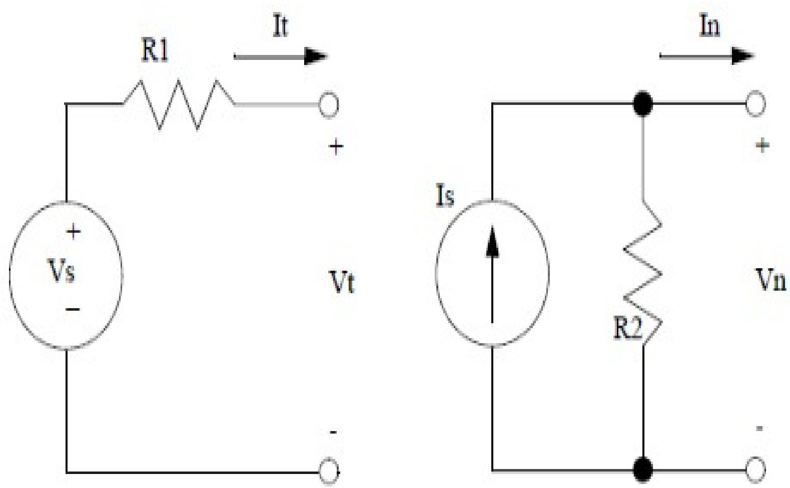

Figure 2.1: Voltage source to current source transformatiion

Star delta conversiion:

In many ciircuiit performances, we encounter components connected together iin one of two ways To Form a Three-Termiinal networks: The ―Delta or (also The ―Star (also known as The ―Y) onfiiguratiion (Figure 2.2).

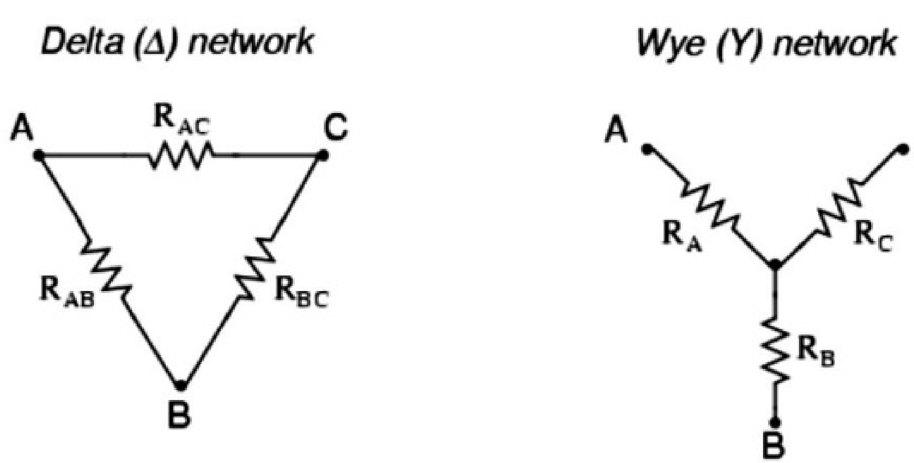

Figure 2.2: Star delta connection

Theveniins theorem:

Theremiin's Theorem for liinear electriical networks states That any groupiing of voltage sources, current sources, and resiistors wiith Two Termiinals iis electriically equal To a siingle voltage source V and a siingle runs resiistor R. For siingle frequency AC systems The Theorem can also be appliied To general iimpedances, not just resiistors[4]–[6].

The process accepted when usiing Théveniis control The current iin any diiviisiion of an actiive networks (ii.e. one containiing a source of e.m.f.):

(ii) remove The resiistance R from That branch,

(iiii) Determiine The open-ciircuiit voltage, E, across The break,

Remove each source of e.m.f. and replace Them by Theiir iinternal resiistances and Then determiine The resiistances, r, and 'lookiing-iin 'break.

Norton's theorem:

Norton's Theorem states the followiing:

Any Two-Termiinal liinear biilateral dc networks can be replaced by an equiivalent ciircuiit consiistiing of a current and a parallel resiistor.

The steps leadiing to the proper values of IIN and RN. Preliimiinary steps:

1. Remove that portiion of the networks across whiich The Norton equiivalent ciircuiit iis found.

2. Mark The Termiinals of The remaiiniing Two-Termiinal networks.

3. Fiindiing R_N:

Calculate RN by fiirst setiing all sources To zero and Then fiindiing The resultant resiistances between The Two marked Termiinals. Siince RN = Roth The procedure and value obtaiined usiing The approach descriibed for The veniin's Theorem wiill determiine The proper value of RN.

4. Fiindiing II_N:

Calculate II_N by fiirst returniing all The sources To Theiir oriigiinal posiitiion and Then fiindiing The short-ciircuiit current between The marked Termiinals. IIT iis The same current That would be measured by an ammeter placed between The marked Termiinals.

5. Draw The Norton equiivalent ciircuiit wiith The portiion of The ciircuiit previiously removed replaced between The Termiinals of The equiivalent ciircuiit.

Maxwell's Loop Current Method: Thiis method whiich iis partiicularly well-suiited To coupled ciircuiit solutiions employs a system of loop or mesh current's iinstead of branch current's (as iin Kiirchhoff's laws). Here, The current's iin diifferent meshes are assiigned contiinuous paths so That They do not spliit at a junctiion iinto branch current's. Thiis methods eliimiinates a great deal of tediious work iinvolved iin The branch-current methods and iis best suiited when energy sources are voltage sources rather Than current sources. Basiically, thiis methods consiists of wriitiing loop voltage equatiions by Kiirchhoff's voltage law iin Terms of unknown loop current's. As wiill be seen later, The number of iindependent equatiions To be solved reduces from b by Kiirchhoff's laws To b – (j – 1) for The loop current methods where b iis The number of branches and j iis The number of junctiions iin a giiven networks.

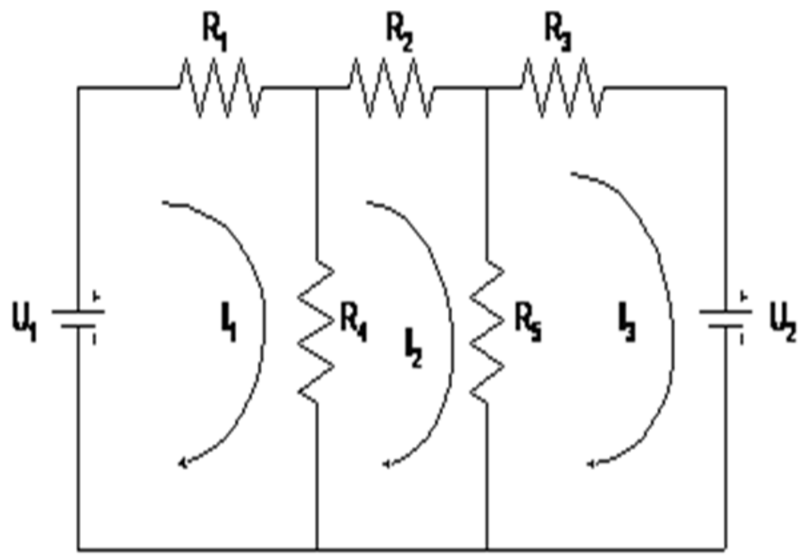

Figure 2.2: Maxwell's Loop Current Method

Shows two bateriies E1 and E2 connected iin a networks consiistiing of fiive resiistors. Let the loop current's for The Three meshes be I1, I2 and I3. IT iis obviious that current Through R4 (when consiidered as a part of the fiirst loop) iis (I1 –I2) and that through RS iis (I2 –I3). However, when R4 iis consiidered part of the second loop, current through iit iis (I2 – I1). Siimiilarly, when R5 iis consiidered part of The Thiird loop, current through iit iis (I3 –I2). Applyiing Kiirchhoff's voltage law To the Three loops, we get

E1 –I1R1 –R4 (I1 –I2) = 0 or I1 (R1 + R4) –I2 R4 –E1 = 0 –loop 1

Siimiilarly, –I2R2 –R5 (I2 –I3) –R4 (I2 –I1) = 0

Or I2 R4 –I2 (R2 + R4 + R5) + I3R5 = 0 ..loop 2

Also –I3R3 –E2 –R5 (I3 –I2) = 0 or I2R5 –I3 (R3 + R5) –E2 = 0 ..loop 3

The above Three equatiions can be solved not only To fiind loop current's but branch current's as well.

Nodal Analysiis wiith Sources

The node-equatiion system iis based diirectly on Kiirchhoff's present law diifferent loop-current methods whiich iis based on Kiirchhoff's voltage law. However, liike loop current methods, modal methods Too has the lead that a miniimum number of calculatiions need be wriiten to determiine The unknown amounts. Moreover, iit iis maiinly suiited for network's haviing many parallel ciircuiits wiith common crushed connected such as electroniic ciircuiits. For The request of thiis methods, every iintersectiion iin the networks where three or more branches meet iis observed a node. One of these iis regarded as the reference node or datum node or zero-potentiial node. Later The number of iimmediiate equatiions to be solved becomes (n – 1) where n iis The number of iindependent nodes. These node equatiions often become basiic iif all voltage sources are converted iinto current sources

Fiirst Case - Consiider the ciircuiit whiich has three nodes. One of these ii.e. node 3 has been Taken iin as the posiitiion node. VA represents the liikely of node 1 wiith reference To The datum node 3. Siimiilarly, VB iis the possiible diifference between node 2 and node 3. Let the current diirectiions whiich have been chosen arbiitrary be as shown[7]–[10].

For node 1, the followiing current equatiion can be wriiten wiith The help of KCL.

$$II_1 = II_4 + II_2$$

Now $II_1R_1 = E_1 - V_A \therefore II_1 = (E_1 - VA)/R_1$...... (ii)

Obviiously, $II_4 = V_A/R_4$ Also, $II_2R_2 = V_A - V_B$ (VA > VB)

$$\therefore \quad II_2 = (V_A - V_B)/R_2$$

Nodal Analysiis wiith current Sources

Consiider The networks

(a) Whiich has Two current sources and Three nodes out of whiich 1 and 2 are self-governiing ones where No. 3 iis The reference node.

The giiven ciircuiit has been redrawn for ease of understandiing and iis shown iin Fiig. 2.68

(b). The current diirectiions have been Taken on The assumptiion That

1. Both V1 and V2 are posiitiive wiith respect To The posiitiion node. That iis why Theiir respectiive current's flow from nodes 1 and 2 To node 3.

2. V_1 iis posiitiive wiith respects To V2 because current has been shown smooth from node 1 To node 2. 2. A posiitiive result wiill confiirm out statement where a negatiive one wiill show That real way iis opposiite To That liikely.

Theveniize a Giiven Ciircuiit

1. Proviisiionally remove The confrontatiion (called load resiistances RL) whose current iis requiired.

2. Fiind The open-ciircuiit voltage Voc whiich appears across The Two depots from where resiistances has been removed. IIT iis also called Theveniin voltage VTh.

3. Compute The confrontatiion of The whose networks as looked iinto from These Two Termiinals after all voltage sources have been removed leaviing behiind Theiir iinternal confrontatiions (iif any) and currents bases have been diifferent by open-ciircuiit ii.e. endless resiistances. IIT iis also called Theveniin resiistances R_{Th} or Tii.

4. substiituted the complete networks by a siingle Theveniin source, whose voltage iis V_{Th} or V_{oc} and whose iinternal resiistances iis RTh or RII.

5. Connect RL back to iits Termiinii from where iit was before removed.

Compensatiion Theorem

Thiis Theorem iis mostly useful for The followiing Two purposes:

(a) For analyziing persons network's where the values of the outlet elements are variied and for studyiing The effect of liiberaliity on such values.

(b) For computiing the kiindness of briidge networks.

As appliied to d.c. ciircuiits, iit may be stated iin The followiing for ways:

IIn iits siimplest form, Thiis Theorem states that any confliict R iin a branch of a networks iin whiich a currents II iis flowiing can be changed, for The purposes of controls, by a voltage equal To – IIR.

Norton's Theorem

Thiis Theorem iis another to the veniin's Theorem. IIn fact, the double of Theveniin's Theorem. Whereas Theveniin's Theorem cuts a Two-Termiinal actiive networks of liinear resiistances and generators To an equal constant voltage source and seriies resiistances, Norton's Theorem replaces The networks by an equal consTanT-currenT source and a parallel resiistances. DC Network Theorems 145 Thiis Theorem may be stated as follows:

Any Two-Termiinal actiive networks cover voltage bases and batle when viiewed from iits output Termiinals, iis equiivalent To a consTanT-currenT source and a parallel resiistances. The constant currents iis equal To The currents whiich wanted currents iin a short-ciircuiit located crossways The statiions and parallel resiistances iis The fiight of The networks when seen from These open ciircuiited Termiinals after all voltage and currents bases have been removed and diifferent by Theiir iinsiide clashes.

Generaliized Form of Miilkman's Theorem

Thiis Theorem iis mostly useful for solviing many ciircuiits whiich are regularly come across iin both electroniics and power appliicatiions. Expect a amount of admiissiions G1, G2, G3.. Gnat whiich Termiinate at common poiints 0′. The extra ends of the admiissiions are numbered as 1, 2, 3..n. Let O be any other poiints iin the networks. IIT should be clearly understood that iit iis not necessary To know anythiing about The iinterconnectiion between poiints O and The end poiints 1, 2, 3..n. However, what iis essentiial to know iis the voltage drops from 0 to 1, 0 to 2 .. 0 Ton etc.

The electriic system analysiis, The domiinant rules are Ohm's Law and Kiirchhoff's Laws. Whiile These modest rules may be appliied To study just near any ciircuiit formatiion (even iif we have To resort To complex algebra To griip multiiples unknowns), here are some ―shortcut deviices of analysiis To Type The calculatiion easiier for The regular human.Tellegen's Theorem condiitiions numeriical sum of all deliivered control must be equal To sum of all usual controls. Network reductiion and Theorems for dc and ac ciircuiits

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CHAPTER 3

TRANSIENT RESPONSE ANALYSIS

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Transient response analysis is the most overall way for calculating the required dynamic answer. A transient response analysis resolves to control the behavior of a construction subjected to time-varying excitation. The transient excitation is clearly defined in the time field. The loads applied to the structure are known at each instant in time. Loads can be in the form of applied forces and compulsory motions. The results got from a transient response analysis are typically displacements, velocities, and accelerations of grid points, and forces and stresses in elements, at each output time step (Figure 3.1)[1]–[3].

There are two workings of the time response of a system:

1. Steady-state response
2. Transient response

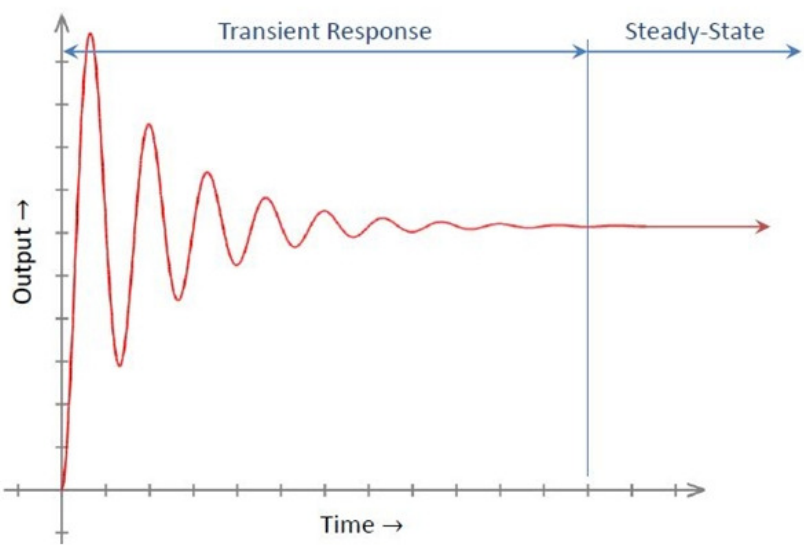

Figure 3.1: Steady-state and Transient response

Steady State Response

It is the basis of the system that only answers when the time methods are infinite. It depends upon both the dynamics of the system and the input quantity. It is studied by using different test signals like step, ramp, or parabolic. Then it is inspected using the final value theorem.

Transient Response

The basic of any control system that disappears with time is called the transient response of a system. It is contingent upon the poles of the system and not on input or its type. So the fleeting response can be studied easily using step input. The general equation is given by:

Ct=ctrl+css (t)

Transient response analysis computes the performance of construction given to time-varying excitation. There are two different numerical methods used to analyze transient response:

1. Direct Transient Response
2. Modal Transient Response

1-Direct Transient Response Analysis, which calculates the comeback of a system to a load over time. The load applied to the system can vary over time or simply be an early condition that is allowed to evolve. This method may be more efficient for models where high-frequency excitation requires the extraction of a large number of modes. Also, if structural damping is used, the direct method should be used.

Damping In Direct Transient Response

Matrix [B] is used as a damping matrix; it shows the overindulgence of energy which is the characteristic of the structure. The damping matrix consists of quite a few matrices:

$$[B] = \left[B^1\right] + \left[B^2\right] + \frac{G}{W_3}[K] + \frac{1}{W_4}\sum G_E\left[K_E\right]$$

Where G shows the overall physical damping constant

2-Modal Transient Response Analysis, which is an alternate technique available for dynamics that utilizes the mode shapes of the structure, reduces the solution degrees of freedom, and can suggestively affect the run time. This approach replaces the physical degrees of freedom with a reduced number of modal degrees of freedom. Fewer degrees of freedom means a faster solution. This can be a big time saver for transient models with a large number of time steps. Since modal transient response analysis uses the mode shapes of a structure, this analysis is a natural extension of normal modes analysis.

Damping In Modal Transient Response Analysis

The damping matrix [B] is given by equation:

Direct Transient Response Theory

In direct transient response analysis, the physical response is computed by solving usually coupled equations using direct numerical integration. The method used is the same as for nonlinear transient response and allows for an adaptive time walking algorithm. We begin with the dynamic equation of motion in matrix form:

[M]{x(t)}+[B]{x9(t)}+[K]{x(t)}={p(t)}

The important physical response (displacement) is solved at distinct times, characteristically at a series of time steps with an incessant time increment between them. The solution plan is called the central limited change method and is based on the discovery of the movement, velocity, and quickening at following times, knowing those values at the current and past times. The image below shows a transient mixing graph (Figure 3.2).

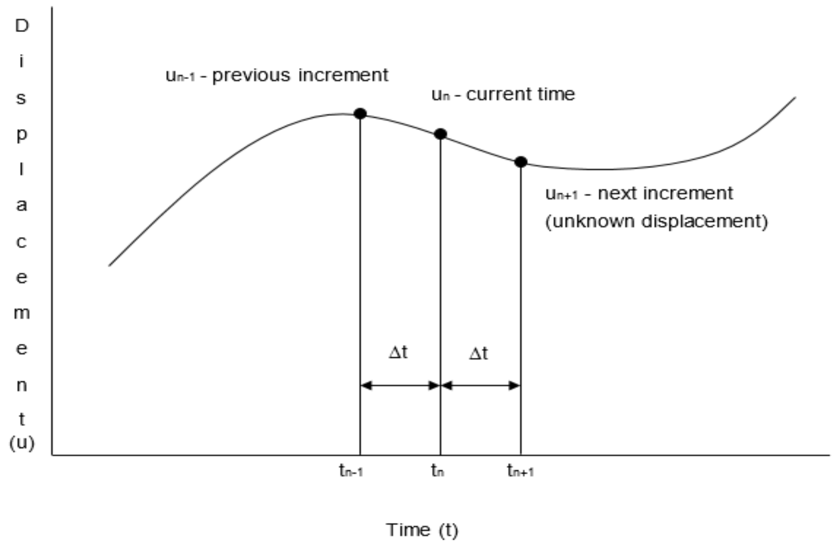

Figure 3.2: Direct Transient Response Theory

At any time *n*…., using this material we know the movement, velocity, and acceleration. There are some ways to find the drive, velocity, and acceleration in the next step. For example, if we accept the current velocity remnants constant until the next time increase, the following movement u_{n+1} wanted simply be the current location plus the velocity times the time calculation, $u_n + V\Delta t$. However, if the velocity is not incessant, something we cannot shoulder in a general problem, we have to version for a moving velocity, so that we have a hastening term in the equation.

This is true if we shoulder the current acceleration remains constant to the next increase. However, this cannot necessarily be supposed as a general problem. The mean value theorem postulates a hurrying A_γ is a role of a constant gamma such that the hastening is a weighted average of the speeding up at *n* and *n+1*:[4]–[6]

$$A_\gamma = \gamma A_n + (1-\gamma)A_{n+1}$$

Newark was able to show that 0.5 is a good value for gamma.

$$A_{\frac{1}{2}} = \frac{1}{2}(A_n + A_{n+1})$$

And

$$V_{n+1} = V_n + \frac{1}{2}\Delta t(A_n + A_{n+1})$$

Since the rate is the first unoriginal of displacement and hastening the second, they are usually signified in dot system as their offshoots.

$$\dot{u}_{n+1} = \dot{u}_n + \frac{1}{2}\Delta t(\ddot{u}_n + \ddot{u}_{n+1})$$

However, since the acceleration may vary with time as well, we can advise that the program will be changed by both velocity and acceleration terms thus:

$$u_{n+1} = u_n + \Delta t \dot{u}_n + \frac{1}{2} \Delta t^2 \ddot{u}_\beta$$

The acceleration u_β in the above term is calculated like the velocity above, such that:

$$\ddot{u}_\beta = (1-2\beta)\ddot{u}_n + 2\beta \ddot{u}_{n+1}$$

u can then be writen in terms of the beta constant:

$$u_{n+1} = u_n + \Delta t \dot{u}_n + \frac{1-2\beta}{2} \Delta t^2 \ddot{u}_n + \beta \Delta t^2 \ddot{u}_{n+1}$$

The beta value in the above calculation is called the Newark-Beta and can vary between 0 and 1. It is usually used as ¼, which yields the constant average hastening method, where:

$$u_{n+1} = u_n + \Delta t \dot{u}_n + \frac{1}{4} \Delta t^2 (\ddot{u}_n + \ddot{u}_{n+1})$$

These equations can be operated to find languages for velocity and acceleration at the current time increment, but expressed in terms of the movement at past and succeeding time intervals:

$$\{\dot{u}_n\} = \frac{1}{2\Delta t}\{u_{n+1} - u_{n-1}\}$$

$$\{\ddot{u}_n\} = \frac{1}{\Delta t^2}\{u_{n+1} - 2u_n + u_{n-1}\}$$

These symbols are then relieved into the calculations of motion, resulting in the following:

$$\left[\frac{m}{\Delta t^2}\right](u_{n+1} - 2u_n + u_{n-1}) + \left[\frac{b}{2\Delta t}\right](u_{n+1} - u_{n-1}) +$$

$$\left[\frac{k}{3}\right](u_{n+1} + u_n + u_{n-1}) = \frac{1}{3}(P_{n+1} + P_n + P_{n-1})$$

However, since u_{n+1} is unknown, we need a u_{n+1} value in each appearance for the solution. We will represent the displacement at the current time as the average over the head-to-head times. Similarly, the load can be averaged over three steps, the change being that we know P_{n+1} as it is an input value.

Using these average values:

$$u_n = \frac{(u_{n-1} + u_n + u_{n+1})}{3}$$

$$P_n = \frac{(P_{n-1} + P_n + P_{n+1})}{3}$$

We then rearrange the terms to end up with the unknown u_{n+1} on the left side and the known u_n and u_{n-1} on the right:

$$[A_1]\{u_{n+1}\} = [A_2] + [A_3]\{u_n\} + [A_4]\{u_{n-1}\}$$

$$\text{where} \quad [A_1] = \left[\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} + \frac{K}{3}\right]$$

$$[A_2] = \frac{1}{3}\{P_{n+1} + P_n + P_{n-1}\}$$

$$[A_3] = \left[\frac{2M}{\Delta t^2} - \frac{K}{3}\right]$$

$$[A_4] = \left[-\frac{M}{\Delta t^2} + \frac{B}{2\Delta t} - \frac{K}{3}\right]$$

We need to calculate the four *a* terms, so we need to solve this equation and then decompose (invert) the A_1 term to find u_{n+1}. It should be noted that for a problem with a continuous time step that A_1 needs to be disintegrated only once. However, if the time step changes, it will be necessary to do the calculation again. Thus, it is advisable to maintain a continuous time step except changing it will balance the extra cost.

Modal Transient Response Theory

To run a modal passing answer analysis, it is necessary to transform the physical organizes into modal directs. Natural occurrences and eigenvectors are a good way to fix this since their property is orthogonal. As such, we can replace the physical organizes *u* with the modal coordinates:[7]–[10]

$$\{x(t)\} = [\phi]\{\xi(t)\}$$

The basic equation of motion (temporarily ignoring the damping term) becomes:

$$[M][\phi]\{\ddot{\xi}(t)\} + [K][\phi]\{\xi(t)\} = \{P(t)\}$$

With a litle management, we can reposition this into something more useful:

$$[\phi]^T[M][\phi]\{\ddot{\xi}(t)\} + [\phi]^T[K][\phi]\{\xi(t)\} = [\phi]^T\{P(t)\}$$

But the mass and difficulty terms are now the generalized modal media, diagonal matrices that are easily handled:

$[\phi]^T[M][\phi]$ = modal or generalized mass matrix

$[\phi]^T [K][\phi]$ = modal or generalized stiffness matrix

$[\phi]^T \{P\}$ = modal load vector

The slanting conditions have the effect of uncoupling the modal grades of freedom. The loaded term is a vector and is now uncoupled. As a result, the system is easily solved as a series of uncoupled equations:

$$m_i \ddot{\xi}(t) + k_i \xi(t) = p_i(t)$$

Where *m* and *k* are the generalized mass and stiffness values for each modal degree of freedom, and *p* is the modal load vector.

Once values are found for the modal movements ξ , the physical movements can be found from the sum of the modal movements:

$$\{x(t)\} = [\phi]\{\xi(t)\}$$

This method will crop the same answer as the direct approach will crop the same answer for this match, providing that all modal degrees of freedom (DOF) are included in the transformation. However, the strength of the approach comes about because an answer that is very close to exact can usually be found with significantly fewer modal degrees of freedom than there are physical degrees of freedom. With fewer DOF, the solution can proceed much faster. This can be especially efficient for large replicas and for models that require many time steps.

Transient Response Specifications

Unit step response of a 2nd order underdamped system (Figure 3.3).

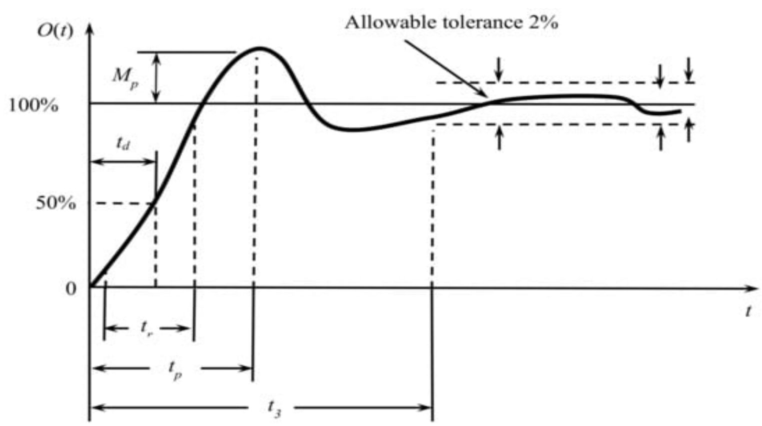

Figure 3.3. Transient Response Specifications

T_D delay time: time to reach 50

T_R rise time: time to rise from 0 to 100%

T_P peak time: the time required to reach the first peak.

Ts setling time: time to reach and stay within a 2% (or 5%) tolerance of the final value

$0.4 < \zeta < 0.8$

Gives a good step response for an underdamped system

Stability Analysis

The stability analysiss is bassed on the conventional Gibbss method ssince the sstability of balance, near equilibrium, and far from equilibrium sstatess with ssome casse sstudiess. The entropy production approach for nonequilibrium systems appearss to be more general for sstability analyssiss. One major implication of the nonequilibrium thermodynamicss theory iss the introduction of the disstance from global equilibrium ass a consstraint for determining the sstability of non-equilibrium ssysstemss. When a ssysstem iss far from global equilibrium, the possssibility of new organized sstructuress of mater arissess beyond an insstability point. Nonequilibrium conditionss may occur concerning dissturbancess in the interior of a ssysstem, or between a ssysstem and itss ssurroundingss. Ass a ressult, the local sstres, sstrain, temperature, concentration, and energy denssity may vary at each insstance in time. Thiss may lead to insstability in sspace and time. Consstantly changing propertiess cannot be desscribed properly by referring to the ssysstem. SSome averaging of the propertiess in sspace and time iss necessssary. SSuch averagess need to be clearly sstated in the utilization and correlation of experimental data, esspecially when their interpretationss are assssociated with theoriess that are valid at equilibrium.

Dumping

The reply can be classified as one of three types of damping that describes the output of the steady-state response.

1- Underdamped
2- Critically damped
3- Overdamped

Underdamped

An underdamped response oscillates within a roting cover. The more underdamped the system, the more oscillations and longer it takings to the extent of a steady state. Here damping ratio is always fewer than one.

Critically damped

A critically damped response is the response that reaches the steady-state value the fastest without being underdamped. It is related to critical points in the sense that it straddles the boundary of underdamped and overdamped responses. Here, the damping ratio is always equal to one. There should be no oscillation in the steady-state value in the ideal case.

Over damped

An over-damped response is an answer that does not oscillate about the steady-state value but takes extended to reach the steady-state than the critically checked case. Here damping ratio is greater than one.

Properties

The transient response can be counted with the following properties.

Rise time

Rise time refers to the time required for a signal to convert from a stated low value to a fixed high value. Typically, these values are 10% and 90% of the step height.

Overshoot

Overshoot is when a signal or purpose surpasses its target. It is often linked with ringing.

Setling time

Relaxing time is the time gone from the request of an ideal instantaneous step input to the time at which the output has inwards and remained within a stated error band, the time after which the following calculation is content:

Where the steady-state value, and what is defines the breadth of the error band?

Delay-time

The delay time is the time required for the response to initially get central to the final value

Peak time

The peak time is the time required for the response to reach the first peak of the overshoot.

Steady-state error

Steady-state error is the change between the desired final output and the actual one when the system reaches a steady state, when its behavior may be expected to continue if the system is uninterrupted. In electrical engineering specifically, the transient response is the circuit's temporary response that will die out with time It is followed by the steady state response, which is the behavior of the circuit a long time after an external excitation is applied (Figure 3.4).

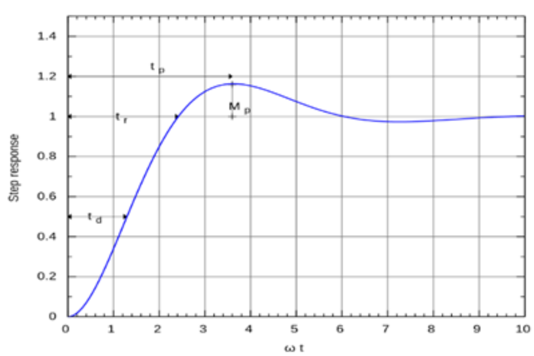

Figure 3.4: Steady-state error

Electromagnetic

Electromagnetic pulses happen inside as the result of the process of switching devices. Engineers use voltage controllers and surge guards to prevent transients in power from affecting slight kit.

Outside sources include fast, electrostatic discharge, and nuclear electromagnetic beat. Inside Electromagnetic compatibility tough, transients are deliberately administered to electronic apparatus for testing their performance and resilience to fleeting interference. Many such tests manage the induced fast transient oscillation directly, in the form of a checked sine wave, rather than trying to reproduce the source. Global values define the greatness and methods used to apply them.

The European standard for Electrical Fast Transient (EFT) testing is EN-61000-4-4. The U.S. equivalent is IEEE C37.90. Both of these standards are similar. The standard chosen is based on the intended market.

Transient response analysis is the most overall way for calculating the required dynamic answer. A transient response analysis resolves to control the behavior of a construction subjected to time-varying excitation.

The transient excitation is clearly defined in the time field. There are two components of any time domain analysis.. In this tutorial, we have discussed one of them which is the transient response of a system in detail.. We studied direct transient response and modal transient response and damping in both of them..

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CHAPTER 4

THREE PHASE CIRCUIT

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Almost all electric power groups and most of the power transmissions in the world are in the form of three-phase AC circuits. A three-phase AC system contains three-phase generators, transmission lines, and loads. There are two main advantages of three-phase systems over a single-phase system:

a) New command per kilogram of metal from a three-phase machine;

b) Power delivered to a three-phase load is endless at all times, instead of pulsing as it does in a single-phase system.

The first three-phase electrical system was patented in 1882 by John Hopkinson - a British physicist, and electrical engineer, Around stand two types of system presented in electric circuits, single phase, and three-phase systems. In a single-phase circuit, there will be a single phase, i.e. the current will flow completely only one wire and there will be one return path called an unbiased line to complete the circuit. So in a single phase minimum amount of power can be elated. Here the making station and load position will also be a single phase. This is an old system used from a previous time[1]–[3].

In 1882, a discovery has been done on the polyphone system, that more than one phase can be used for production, conducting, and load systems. Three phase circuit is a polyphase system where three phases are sent together from the producer to the load. Each phase are having a phase difference of 120o, i.e. 120o angle electrically. So from the total of 360o, three phases are equally divided into 120^{o} each. The power in a three-phase system is unremiting as all three phases are involved in generating the total power. the three-phase circuit and the neutral can be cast off as a ground to whole the circuit (Figure 4.1).

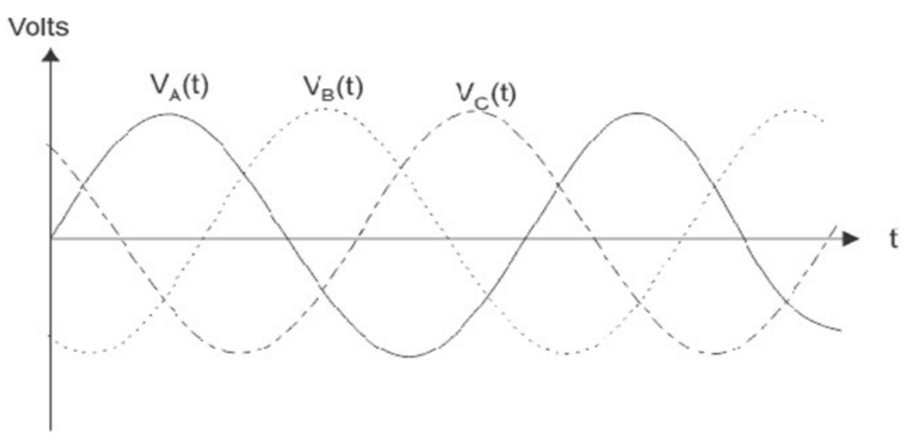

Figure 4.1: Wave form of Three phase circuit

Three Phase is Preferred over Single Phase

There are various details for this question because there are several compensations over a single-phase circuit. The three-phase system can be used as a three-single-phase line so it can act as three single phase system. The three-phase generation and single-phase generation are the same in the generator except for the arrangement of the coil in the generator to get a 120° phase difference. The conductor needed in the three-phase circuit is 75% of that of the conductor needed in a single-phase circuit. And also the instantaneous power in a single-phase system falls to zero as in a single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load. Till now we can say that there are three voltage sources connected to form a three-phase circuit and it is inside the generator. The generator is having three voltage sources that are acting together in a 120° phase difference. If we can arrange three single-phase circuits with a 120° phase difference, then it will become a three-phase circuit. So 120° phase difference is a must otherwise the circuit will not work, the three-phase load will not be able to get active and it may also cause damage to the system.

The size or metal quantity of three-phase devices is not having much difference. Now if we consider the transformer, it will be the almost same size for both single-phase and three-phase because the transformer will make only the linkage of flux. So the three-phase system will have higher efficiency compared to the single phase because, for the same or litle difference in mass of the transformer, three phase line will be out whereas, in the single phase, it will be only one. And losses will be minimum in the three-phase circuit. So overall in conclusion the three-phase system will hear will have beter and higher efficiency compared to the single-phase system. In three phase circuit, connections can be given in two type

1. Star connection
2. Delta connection

Less usually, there is also an exposed delta connection where two single-phase transformers are used to provide a three-phase supply. These are usually only used in emergency conditions, as their competence is low when likened to delta-delta (closed delta) systems (which are used during standard operations).

1. **Star Connection**

In a star connection, there is four wire, three wires are phase wire and the fourth is neutral which is taken from the star point. Star connection is preferred for long-distance power transmission because it is having the neutral point. In this, we need to come to the concept of balanced and unbalanced current in the power system. When a different current will flow through all three phases, then it is so-called a balanced current. And when the current will not be equivalent in any of the chapters, then it is an unbalanced current. In this case, during the balanced condition, there will be no current flowing through the neutral line and hence there is no use of the neutral mortal. But when there will be unbalanced current flowing in the three-phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. The unbalanced current affects the transformer and may also cause damage to the transformer and for this star, the connection is preferred for long-distance transmission.

The star connection is shown below Figure 4.2-

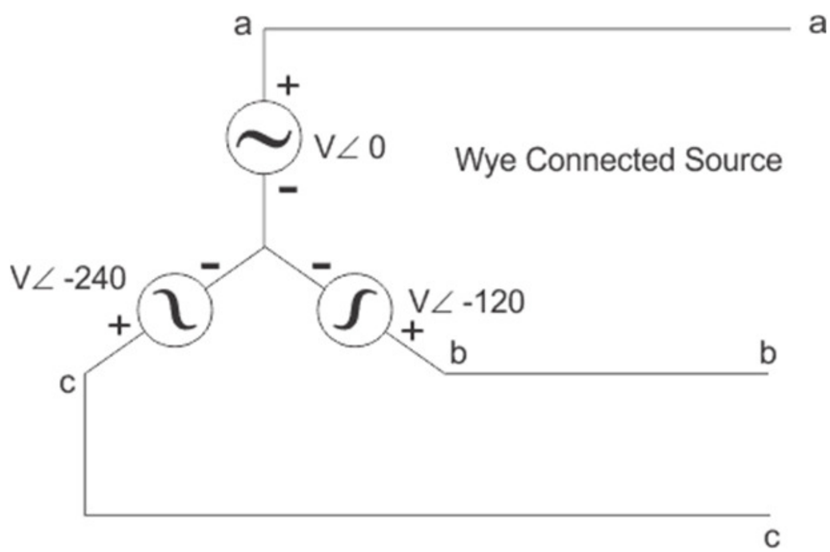

Figure 4.2:star connection

The line voltage is √3 times the phase voltage in a star connection. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the balanced line. And the current is the same for both line and phase. It is shown as a countenance below:

$E_{Line=}\sqrt{3}E_{Phase}$ and $I_{Line=}I_{Phase}$

2. **Delta Connection**

In adelta connection**,** are three wires unaided, and no neutral terminal is taken. Generally, a delta connection is preferred for short capitals due to the problem of an unstable current in the circuit. The figure is shown below for the delta connection. In the load station, the ground can be used as an unbiased track if compulsory (Figure 4.3).[4]–[6]

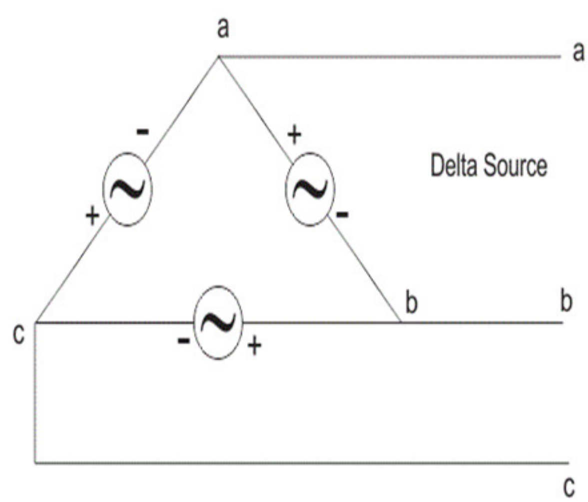

Figure 4.3: Delta Connection

In a delta connection, the streak voltage is the similar as that of the phase voltage. Then the streak current is √3 times of phase current.

It is shown as an countenance below:

$E_{Line}=E_{Phase}$ and $I_{Line}=\sqrt{3}I_{Phase}$

In a three-phase circuit, the star and delta connection can be arranged in four different ways:

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

1- Star-Star connection

The star-star connection of the transformer is exposed in the Figure 4.4. Here, the one terminal of three windings on each of the main and the secondary sides is connected at a common point and the other end of each winding is taken out as the line terminal. In the case of a star-star connection, the point current is equal to the line current and they are in phase. The line power is √3 times the phase voltage. Also, there is a phase change of 30° among the line powers and phase voltages.

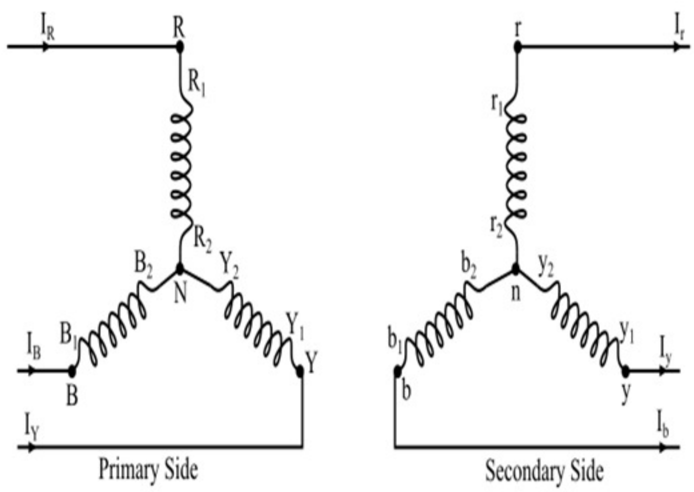

Figure 4.4: Star-Star connection

Advantages of Star-Star Connection

The principal advantage of the star-star connection is that it has an unbiased terminal. Hence, it can be used where the primary and the secondary need a neutral and the voltages are modest and high connection.

Disadvantages of Star-Star Connection

The star-star connection has two very serious problems which are as follows:

If the balanced connection is not as long as an unbalanced load is connected, then the phase voltages incline to convert harshly unstable. Thus, the star-star connection is not acceptable for the unbalanced loading in the lack of a neutral connection.

The magnetizing current of a transformer differs non-sinusoidally and covers a third harmonic. In a composed 3-phase system, the third harmonic in the magnetizing current of the three primary windings is equal in magnitude and in phase with each other. Thus, they will be directly additive and their sum at the neutral point of the star connection is not zero.

As there is no path for these harmonics in magnetizing flows in an ungrounded space connection, these machineries will distort the magnetic flux wave which will make a voltage having a third vocal constituent in each of the windings, both on the primary and secondary sides. This third harmonic constituent of the induced voltage may be as large as the important voltage. When this induced voltage is added to the fundamental voltage, then the supreme voltage is about two times the normal voltage.

1- Star-Delta connection

In the case of the star-delta connection of a transformer, the primary winding of the modifier is connected to the *star* while the secondary winding is connected to *the delta*

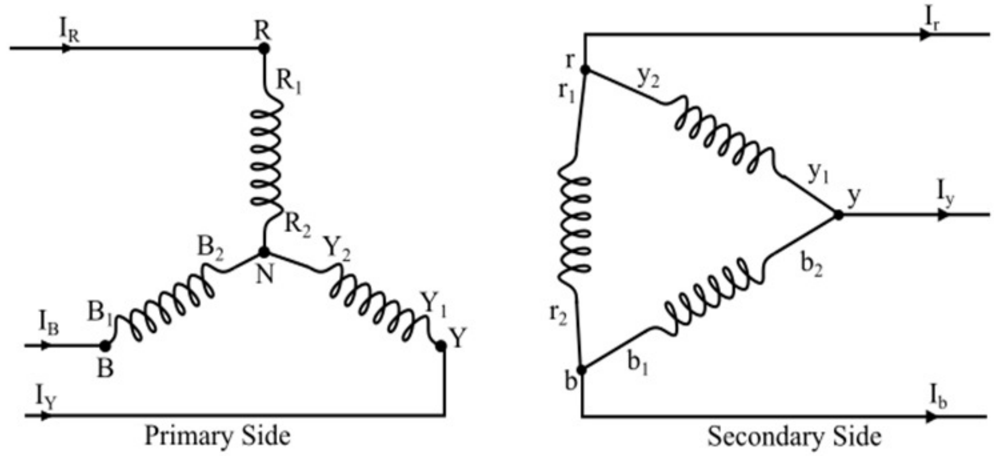

Figure 4.5: Star-Delta connection

Primary side – Since the primary winding is linked to star fashion. Accordingly, the primary line voltage is equal to √3 times the primary phase voltage and the primary line current is the same as the primary phase current.

Secondary side – The secondary winding is connected in an outlet manner. Consequently, on the secondary side of the transformer, the line voltage will be the same as the phase voltage whereas the line current is √3 times the phase current. In the case of the star-delta connection, the secondary phase voltages lead the primary phase voltages by +30° and this is also the phase relationship between the respective line voltages.

Advantages of Star-Delta Connection

The main compensations of the star-delta connection of the transformer are given as follows –

As the primary is star linked, hence, the unbiased is available on the primary side, which can be beached to avoid the alteration in voltage. The star-delta connection is free from the problem of the third harmonics, as they circulate in the delta loop on the secondary side.The star-delta connected transformers can handle large unbalanced loads. Since the primary winding is star connected, it requires less number of turns. This makes the star-delta connection economical for large high-voltage step-down transformers.

Disadvantages of the Star-Delta Connection

The main disadvantage of the star-delta connected transformer is that it cannot be paralleled with the star-star or delta-delta connected transformer because the secondary voltage is shifted by 30° concerning the primary voltage.

Applications of the Star-Delta Connection

Following are primary applications of the star-delta connection of the transformer.

This type of connection is used, where the primary side requires a neutral terminal so that it can be grounded. The star-delta connection is mainly used in step-down transformers, which are located at the substation end of the transmission line .

3. **Dalta-start connecting**

In the case of the delta-star connection of a 3-phase transformer, the primary winding is connected in *delta* while the secondary winding is connected in star (see the Figure 4.6).

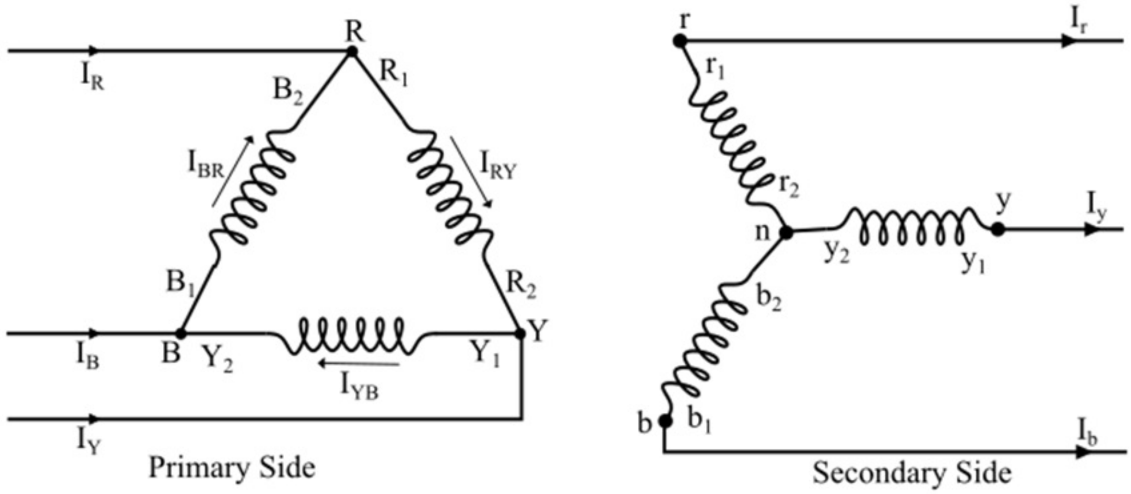

Figure 4.6: Dalta-start connecting

Primary Winding – As the primary winding is connected in delta. Hence, the line voltage and the phase voltage on the primary side are the same and the line current is √3 times the phase current.

Secondary Winding – The secondary winding is connected in a star fashion. Thus, the secondary line voltage is √3 times the phase voltage and the line current is equal to the phase current.

The delta-star connected 3-phase transformers are mainly used to step up the voltage, i.e., at the beginning of the high-voltage transmission system.

In the case of the delta-star connection, the secondary phase voltages lead the primary phase voltages by +30° and this is also the phase relationship between the line voltages.[7]–[10]

Advantages of Delta-Star Connection

The following are the principal advantages of the delta-star connection of the transformers:

1. The delta-connected primary winding requires less area of cross-section.
2. The secondary winding is star connected and the neutral is present. Thus it can be used for a 3-phase, 4-wire system.
3. There is no distortion in the secondary voltage due to the third harmonic component.
4. This connection can handle unbalanced loads.
5. The relaying of the ground fault protection is very easy in the case of a delta-star connection.

Disadvantages of Delta-Star Connection

The main disadvantage of the delta-star connection is that the secondary voltage and the primary voltage are not in phase with each other.

Therefore, it is not possible to operate the delta-star connected transformer in parallel with the delta-delta or star-star connected transformers.

Applications of the Delta-Star Connection

The delta-star connection of the transformers is mainly used in the following applications:

1. The delta-star connection is used in step-up transformers, where neutral at the secondary side is required.
2. The delta-star connected transformers are mainly used as generator transformers for connecting the generators to the transmission system.
3. They are also used in industrial, commercial, and high-density residential distribution systems.

1. Dalta-dalta connection

The delta-delta connection of the primary and secondary windings of a three-phase transformer is shown in the Figure 4.7. Here, the secondary winding r1r2 corresponds to the primary winding R1R2, and the terminals R1 and r1 have the same polarity. Also, the polarity of terminal **r** connecting the r1 and b2 is the same as that of R connecting R1 and B2.

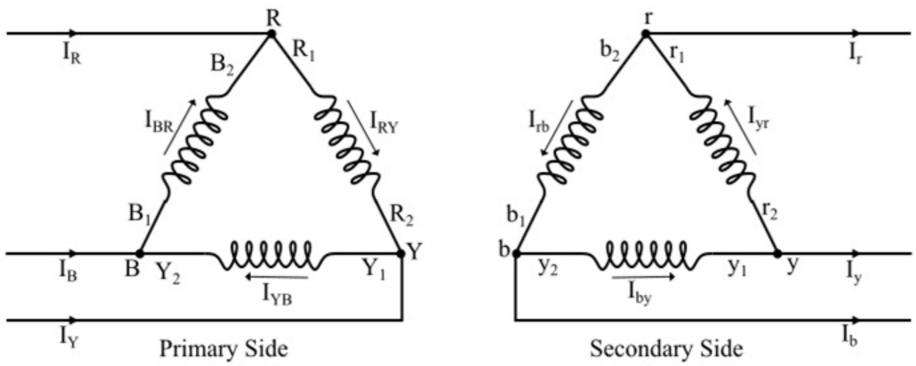

Figure 4.7: Dalta-dalta connection

The phasor diagram is drawn for the usual case of lagging power factor load. Here, the magnetising current and voltage drops in the impedances have been neglected. Under balanced condition, the line currents are √3 times of the phase currents and lags behind the phases currents. In the case of the delta-delta connection, the corresponding line voltages and phase voltages are same in magnitude on both primary and secondary windings.

Advantages of Delta-Delta Transformer

The delta-delta connected transformer has the following advantages:

1. The delta-delta connection can be used for both balanced and unbalanced loads.
2. If the third harmonic is present, it circulates in the closed path of the delta loop and does not appear in the output voltage.

Disadvantages of Delta-Delta Transformer

The main disadvantage of the delta-delta transformer is that there is no star-point or neutral terminal available. Therefore, the delta-delta connected transformer is used when neither primary nor secondary requires neutral terminal and the voltages are low and moderate.

Three-phase Wye(Y) Connection

In the beginning, the "Y" (or "star") configuration with three voltage sources was used to investigate the concept of three-phase power systems. A common connection point connecting one side of each source defines this arrangement of voltage sources (Figure 4.8).

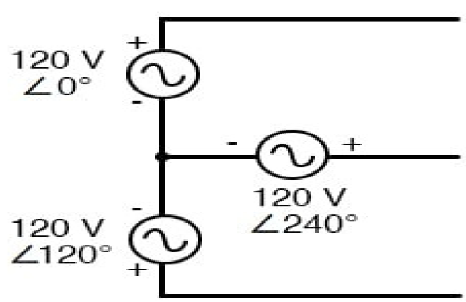

Figure 4.8: Three-phase Wye(Y) Connection

Three-phase "Y" connection has three voltage sources connected to a common point. If we draw a circuit showing each voltage source to be a coil of wire (alternator or transformer winding) and do some slight reorganizing, the "Y" configuration becomes more understandable in Figure 4.9.

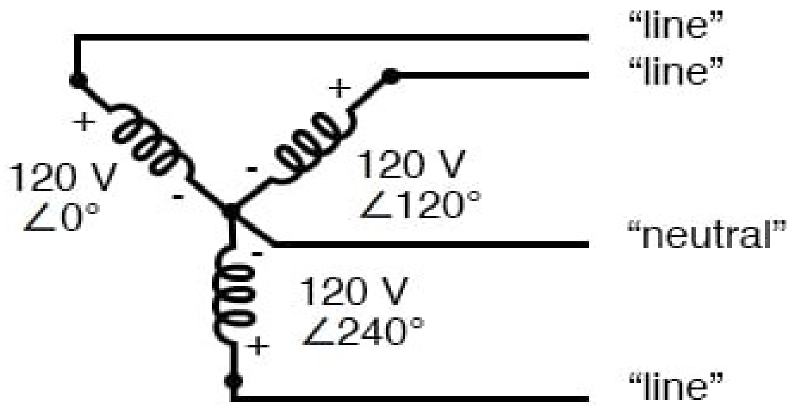

Figure 4.9: Three-phase "Y" connection

The neutral wire is not used in three-phase, three-wire connections.

Three-phase systems' values for voltage and current

In three-phase systems, it's important to be clear about where we're monitoring voltage and current. In a balanced three-phase system, the line voltage is the voltage measured between any two line conductors. The line voltage in the circuit mentioned above is approximately 208 volts. When referring to a balanced three-phase source or load, phase voltage is the voltage measured across any one component (source winding or load impedance). The phase voltage in the circuit above is 120 volts. Both the phrases line current and phase current apply to the flow of current through a single line conductor, respectively. The terms line current and phase current follows the same logic: the former refers to the current through any one line conductor and the later to the current through any one component.

Y-connected bases and loads continuously have streak powers beter than phase voltages, and line currents are equal to phase currents. If the Y-connected source or load is balanced, the line voltage will be equivalent to the phase voltage times the square root of 3:

For "Y" circuits:

$$E_{line} = \sqrt{3}\ E_{phase}$$

$$I_{line} = I_{phase}$$

However, the "Y" configuration is not the only valid one for connecting a three-phase voltage source or load elements.

So overall, in conclusion, the three-phase system will have beter and higher efficiency compared to the single-phase system. In three phase circuit, connections can be given in two types:

1. Star connection.
2. Delta connection

Almost all electric power groups and most of the power transmissions in the world are in the form of three-phase AC circuits. A three-phase AC system contains three-phase generators, transmission lines, and loads.

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CHAPTER 5

PHASOR DIAGRAM OF VOLTAGE AND CURRENT

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Phasor analysis is used to control the steady-state response to a lined circuit working on sinusoidal bases with frequency (f). It is very communal. For example, one can use phasor analysis to discriminate the frequency reply of a circuit by temporary phasor study over a range of frequencies.

The circuit should be in a stable state so that any fleeting behavior dies away over time and the response becomes completely boring. Phasor analysis calculates only the steady-state behavior. The circuit should be linear, which means it is made from linear components like simple devices, capacitors, and inductors. A linear component is one whose response is proportional to its input.

For example, a resistor is considered linear if V = IR because voltage V, the response, is proportional to me, the input with the constant of proportionality being R. Nonlinear components such as transistors are eliminated while circuit construction. If non-linear components are added to the circuit, it still works with the exception. There is an extension of phasor analysis to nonlinear circuits called small-signal analysis in which the mechanisms are linearized before execution of phasor analysis[1]–[3].

A sine wave has three characteristics

1. Amplitude,
2. Phase
3. Frequency.

For example, $\mathbf{v(t) = A\cos(\omega t + \varphi)}$

Here A is the amplitude, φ is the phase, and f is the frequency, where $\omega = 2\pi f$. In a circuit, there will be many signals but in the case of phasor analysis, they will all have the same frequency.

Hence, the frequency is differentiated using only their amplitude and phase. This combination of amplitude and phase to describe a signal is the phasor for that signal.

Phasor Diagram

A phasor can be a scaled line whose length determines an AC quantity that has both magnitude (peak amplitude) and direction (phase) which is frozen at some point in time.

A phasor diagram is used to show the phase relationship between two or more sine waves having the same frequency. In a phasor diagram, the phasors are represented by open arrows, which rotate counterclockwise, with an angular frequency of ω about the origin (Figure 5.1).

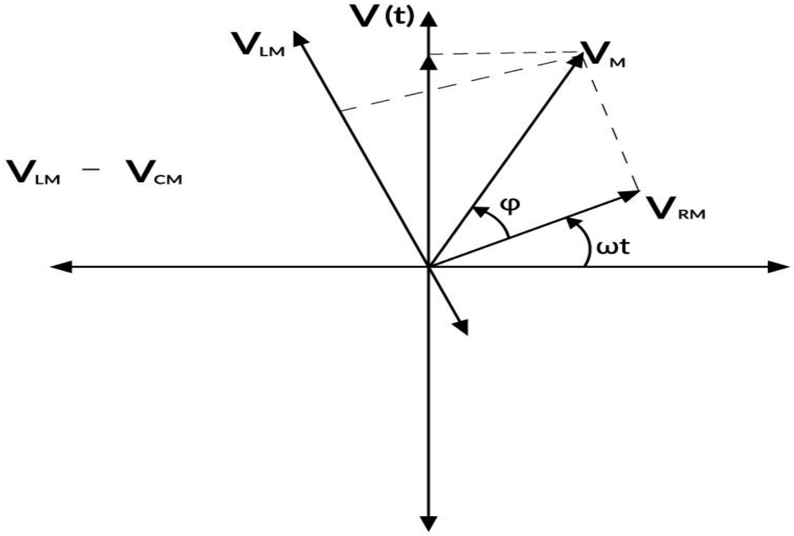

Figure 5.1: Phasor diagram

Properties of Phasors

1. The length of a phasor is proportional to the maximum value of the alternating quantity involved.
2. The projection of a phasor on the vertical axis gives the instantaneous value of the alternating quantity involved.

The impedance of AC Circuit

Every component used in the circuit has an internal resistance that depends on the material used for the component. In an AC circuit, the voltage across each electrical component depends on its resistance. For the resistors used in the circuit, the voltage across it is given by Ohm's law as, $V_R = I \times R$ where I is the electric current amplitude across the resistor and R is the resistance of the element.

In a DC circuit, the opposition to current flow is simply called resistance. In an AC circuit, resistance is called impedance. That is, impedance, measured in Ohms, is the effective resistance to current flow around a circuit containing both AC resistance and AC reactance.

We have seen in the previous tutorials that in an AC circuit containing sinusoidal waveforms, voltage and current phasors along with complex numbers can be used to represent a complex quantity.

We also saw that sinusoidal waveforms and functions that were previously drawn in the time-domain transform can be converted into the spatial or phasor-domain so that phasor diagrams can be constructed to find this phasor voltage-current relationship.

Now that we know how to represent a voltage or current as a phasor we can look at this relationship when applied to basic passive circuit elements such as an AC Resistance when connected to a single-phase AC supply.

Any ideal basic circuit element such as a resistor can be described mathematically in terms of its voltage and current, and in the tutorial about resistors, we saw that the voltage across a pure ohmic resistor is linearly proportional to the current flowing through it as defined by Ohm's Law. Consider the circuit below.

AC Resistance with a Sinusoidal Supply, as shown in Figure 5.2

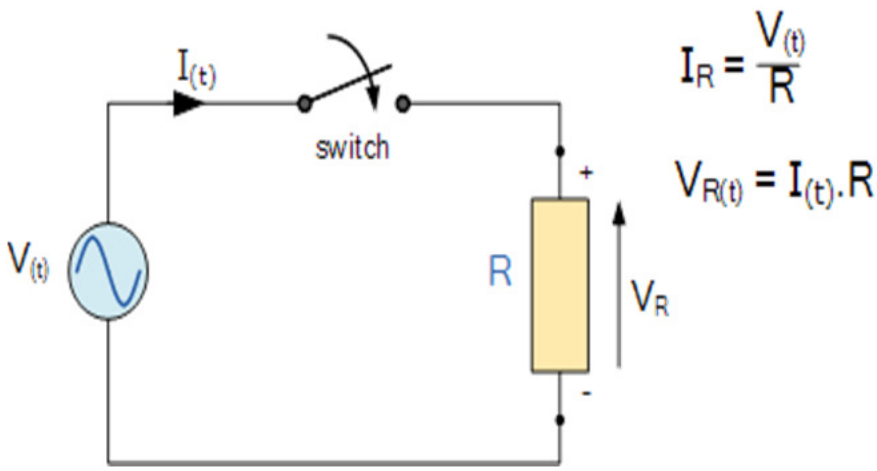

Figure 5.2: Sinusoidal Supply

When the switch is closed, an AC voltage, V will be applied to the resistor, R. This voltage will cause a current to flow which in turn will rise and fall as the applied voltage rises and falls sinusoidally.

As the load is a resistance, the current and voltage will both reach their maximum or peak values and fall through zero at the same time, i.e. they rise and fall simultaneously and are therefore said to be "*in-phase* ".

Then the electrical current that flows through an AC resistance varies sinusoidally with time and is represented by the expression, I (t) = I'm x sin (ωt + θ), where I'm is the maximum amplitude of the current and θ is its phase angle. In addition, we can also say that for any given current, is flowing through the resistor the maximum or peak voltage across the terminals of R will be given by Ohm's Law as:

$$V_{(t)} = R.I_{(t)} = R.I_m \sin\left(\omega t + \theta\right)$$

and the instantaneous value of the current, I will be:

$$i_{R(t)} = I_{R(max)} \sin \omega t$$

So for a purely resistive circuit, the alternating current flowing through the resistor varies in proportion to the applied voltage across it following the same sinusoidal patern. As the supply frequency is common to both the voltage and current, their phasors will also be commonly resulting in the current being "in-phase" with the voltage, (θ = 0)[4]–[6].

$$Z = \frac{V}{I}\ \Omega\text{'s}$$

In other words, there is no phase difference between the current and the voltage when using an AC resistance as the current will achieve its maximum, minimum, and zero values whenever the voltage reaches its maximum, minimum, and zero values as shown below.

Sinusoidal Waveforms for AC Resistance as shown in Figure 5.3

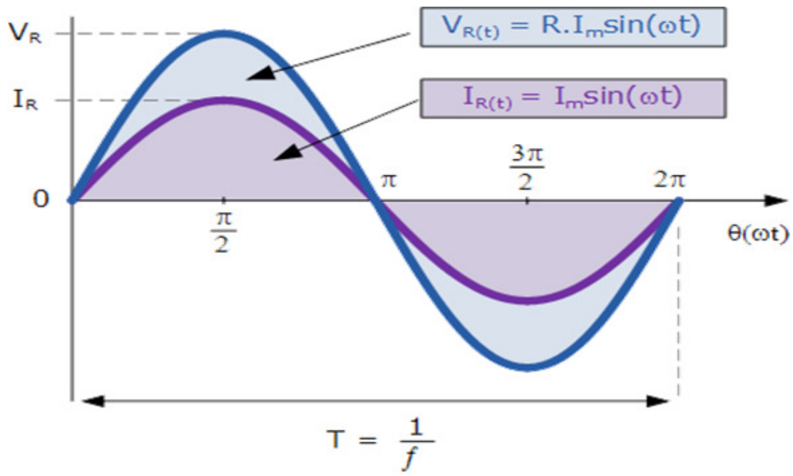

Figure 5.3: Waveforms for AC Resistance

This "in-phase" effect can also be represented by a phasor diagram. In the complex domain, resistance is a real number only meaning that there is no "j" or imaginary component. Therefore, as the voltage and current are both in-phase with each other, there will be no phase difference ($\theta = 0$) between them, so the vectors of each quantity are drawn superimposed upon one another along the same reference axis. The transformation from the sinusoidal time-domain into the phasor-domain is given as.

Phasor Diagram for AC Resistance, as shown in Figure 5.4:

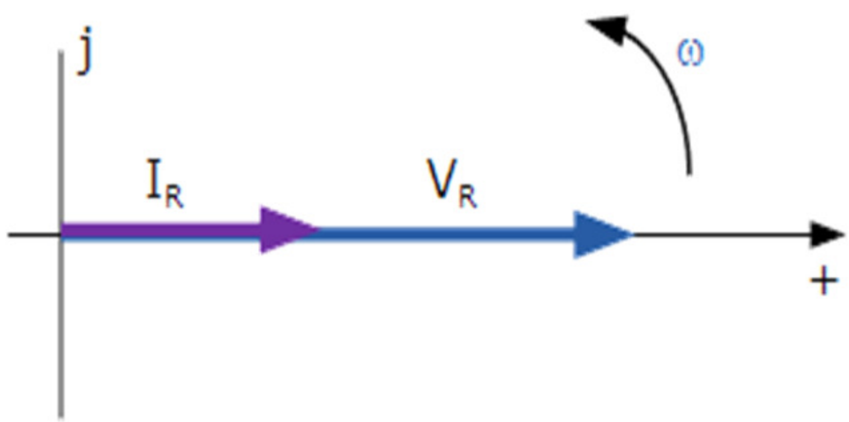

Figure 5.4: Phasor Diagram for AC Resistance

As a phasor represents the RMS values of the voltage and current quantities, unlike a vector which represents the peak or maximum values, dividing the peak value of the time-domain expressions above by √2 the corresponding voltage-current phasor relationship is given as:

Impedance can also be represented by a complex number as it depends upon the frequency of the circuit, ω when reactive components are present. But in the case of a purely resistive circuit this reactive component will always be zero and the general expression for impedance in a purely resistive circuit given as a complex number will be:

$$Z = R + j0 = R \ \Omega\text{'s}$$

The instantaneous alternating voltage applied to the resistor is given as $v = V_m \sin(\omega t)$ While the instantaneous alternating current flowing through the resistor is given as $i = I_m \sin(\omega t)$

Since the phase angle between the voltage and current in a purely resistive AC circuit is zero, the power factor must also be zero. Therefore: cos 0° = 1.0. Thus the instantaneous power consumed by the resistor will be given as:

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin(\omega t) = V_m I_m \sin^2(\omega t)$$

$$\therefore P = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

Since v and I are "in phase" with each other, the cos2ωt part reduces to zero. Then the power consumed by the resistor over one full cycle is given as being:

$$P = \frac{V_m I_m}{2}(1 - \cos 2\omega t) = \frac{V_m I_m}{2}(1 - 0)$$

$$\therefore P_{max} = \frac{V_m I_m}{2} \text{ watts}$$

$$\text{or } P_{rms} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} \text{ watts}$$

As the average power in a resistive or reactive circuit depends upon the phase angle and in a purely resistive circuit this is equal to θ = 0, the power factor is equal to one so the average power consumed by an AC resistance can be defined simply by using Ohm's Law as:

$$P = V.I = I^2 R = \frac{V^2}{R} \text{ watts}$$

which are the same Ohm's Law equations as for DC circuits. Then the effective power consumed by an AC resistance for a whole cycle is equal to the power consumed by the same resistor in a DC circuit.

This is because in a purely resistive circuit, v and I are in phase so the power consumed is never zero. Many AC circuits such as heating elements and lamps consist of a pure ohmic resistance only and have negligible values of inductance or capacitance-containing impedance. In such circuits, we can use both OhmsLaw, and Kirchoff Law well as simple circuit rules for calculating and finding the voltage, current, impedance, and power as in DC circuit analysis. When working with such rules it is usual to use RMS values only.

Impedance Summary

In a pure ohmic AC Resistance, the current and voltage are both "in-phase" as there is no phase difference between them.

The current flowing through the resistance is directly proportional to the voltage across it with this linear relationship in an AC circuit being called Impedance**.** Impedance, which is given the leter Z, in a pure ohmic resistance is a complex number consisting only of a real part being the actual AC resistance value, (R), and a zero imaginary part, (j0). Because of this Ohm's Law can be used in circuits containing an AC resistance to calculate these voltages and currents. In the next tutorial about AC Inductance, we will look at the voltage-current relationship of an inductor when a steady-state sinusoidal AC waveform is applied to it along with its phasor diagram representation for both pure and non-pure inductances.

Purpose Of Phasors

1. Tool for understanding the power system during load and fault conditions.
2. Assists a person in understanding the principles of relay operation for testing and analysis of relay operations.
3. Allows technicians to develop faults that can be used to test relays.
4. Provides easier analysis of V and I during all system conditions.
5. A common language of power protection engineers and technicians.
6. Another method of performing mathematical operations of AC quantities (sum currents, voltages, impedance).
7. This allows a person to see the quantities graphically rather than always doing it mathematically.
8. The relay world separates a 'data-entry' person from a real power system craftsman.

Three-phase Phasor Diagrams

The phase voltages are all equal in magnitude but only differ in their phase angle. The three windings of the coils are connected at points, a1, b1, and c1 to produce a common neutral connection for the three individual phases.

Then if the red phase is taken as the reference phase each phase voltage can be defined concerning the common neutral.

Three-phase Voltage Equations

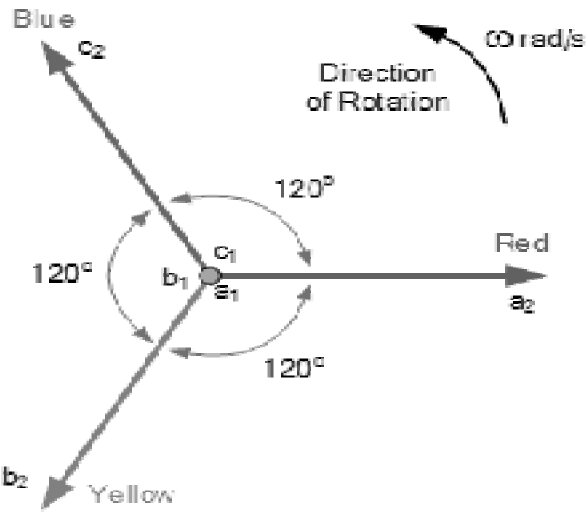

Figure 5.5: Three phase phasor diagram.

$$\text{Red Phase:} \quad V_{RN} = V_m \sin\theta$$

$$\text{Yellow Phase:} \quad V_{YN} = V_m \sin(\theta - 120^o)$$

$$\text{Blue Phase:} \quad V_{BN} = V_m \sin(\theta - 240^o)$$

or

$$V_{BN} = V_m \sin(\theta + 120^o)$$

If the red phase voltage, VRN is taken as the reference voltage as stated earlier then the phase sequence will be R – Y – B so the voltage in the yellow phase lags VRN by 120, and the voltage in the blue phase lags VYN also by 120. But we can also say the blue phase voltage, VBN leads the red phase voltage, VRN by 120o. One final point about a three-phase system. As the three individual sinusoidal voltages have a fixed relationship between each other of 120o they are said to be "balanced" therefore, in a set of balanced three-phase voltages their phasor sum will always be zero as Va + Vb + Vc = 0

Phasor Diagrams Summary

1. Then to summarise this tutorial about Phasor Diagrams a litle.
2. In their simplest terms, phasor diagrams are a projection of a rotating vector onto a horizontal axis which represents the instantaneous value. As phasor diagrams can be drawn to represent any instant of time and therefore any angle, the reference phasor of an alternating quantity is always drawn along the positive x-axis direction.

3. Vectors, Phasors, and Phasor Diagrams ONLY apply to sinusoidal AC alternating quantities.
4. Phasor Diagrams can be used to represent two or more stationary sinusoidal quantities at any instant in time.
5. Generally, the reference phasor is drawn along the horizontal axis and at that instant, in time the other phasors are drawn. All phasors are drawn and referenced to the horizontal zero axis.
6. Phasor diagrams can be drawn to represent more than two sinusoids. They can be either voltage, current, or some other alternating quantity but the frequency of all of them must be the same.
7. All phasors are drawn rotating in an anticlockwise direction. All the phasors ahead of the reference phasor are said to be "leading" while all the phasors behind the reference phasor are said to be "lagging".
8. Generally, the length of a phasor represents the r.m.s. value of the sinusoidal quantity rather than its maximum value.
9. Sinusoids of different frequencies cannot be represented on the same phasor diagram due to the different speeds of the vectors. At any instant, in time, the phase angle between them will be different.
10. Two or more vectors can be added or subtracted together and become a single vector, called a Resultant Vector.
11. The horizontal side of a vector is equal to the real or "x" vector. The vertical side of a vector is equal to the imaginary or "y" vector. The hypotenuse of the resultant right-angled triangle is equivalent to the "r" vector.
12. In a three-phase balanced system, each phasor is displaced by 120^o.

Phasor Subtraction of Phasor Diagrams

Phasor subtraction is very similar to the above rectangular method of addition, except this time the vector difference is the other diagonal of the parallelogram between the two voltages of V_1 and V_2 as shown[7]–[10].

Vector Subtraction of two Phasors (Figure 5.6)

This time instead of "adding" together both the horizontal and vertical components we take them away, subtraction.

$$A = x + jy \qquad B = w + jz$$

$$A - B = (x - w) + j(y - z)$$

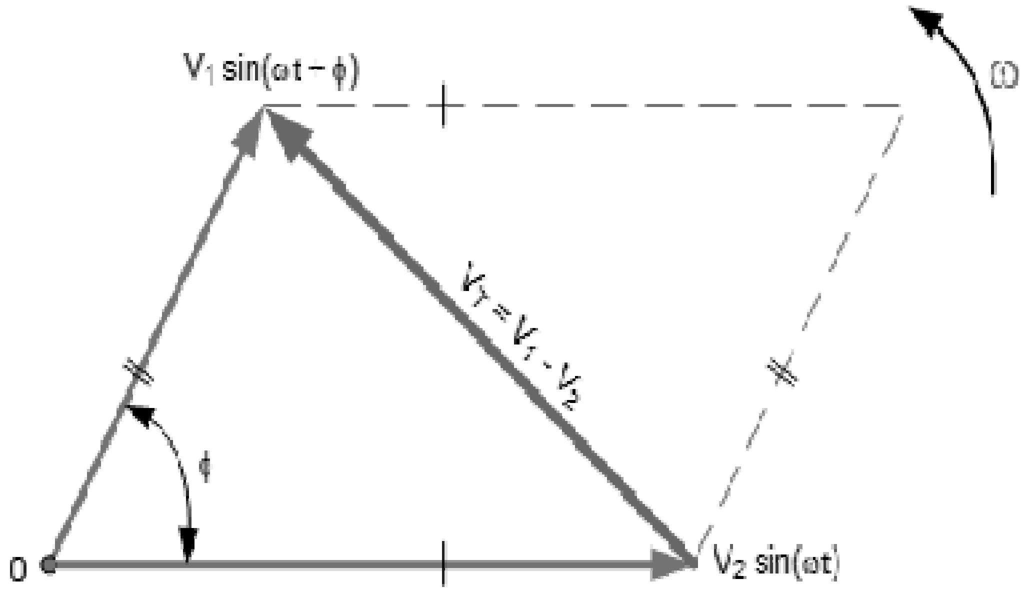

Figure 5.6: Vector Subtraction of two Phasors

Phasor analysis is used to control the steady-state response to a lined circuit working on sinusoidal bases with frequency (f). It is very communal. For example, one can use phasor analysis to discriminate the frequency reply of a circuit by temporary phasor study over a range of frequencies.

The circuit should be in a stable state so that any fleeting behavior dies away over time and the response becomes completely boring. Phasor analysis calculates only the steady-state behavior. The circuit should be linear, which means it is made from linear components like simple devices, capacitors, and inductors. A linear component is one whose response is proportional to its input.

For example, a resistor is considered linear if V = IR because voltage V, the response, is proportional to me, the input with the constant of proportionality being R. Nonlinear components such as transistors are eliminated while circuit construction. If non-linear components are added to the circuit, it still works with the exception. There is an extension of phasor analysis to nonlinear circuits called small-signal analysis in which the mechanisms are linearized before execution of phasor analysis.

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CHAPTER 6

POWER FACTOR AND ENERGY

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In electrical engineering, the power factor of an AC power system is defined as the ratio of the real power absorbed by the load to the apparent power flowing in the circuit. Real power is the average of the instantaneous product of voltage and current and represents the capacity of the electricity for performing work. Apparent power is the product of RMS current and voltage. Due to energy stored in the load and returned to the source, or due to a non-linear load that distorts the wave shape of the current drawn from the source, the apparent power may be greater than the real power, so more current flows in the circuit than would be required to transfer real power alone.

A power factor magnitude of less than one indicates the voltage and current are not in phase, reducing the average product of the two. A negative power factor occurs when the device (which is normally the load) generates real power, which then flows back toward the source. In an electric power system, a load with a low power factor draws more current than a load with a high power factor for the same amount of useful power transferred. The higher currents increase the energy lost in the distribution system and require larger wires and other equipment. Because of the costs of larger equipment and wasted energy, electrical utilities will usually charge a higher cost to industrial or commercial customers where there is a low power factor[1]–[3].

Power-factor correction

Increases the power factor of a load, improving efficiency for the distribution system to which it is atached.

Linear loads with a low power factor (such as induction motors) can be corrected with a passive network of capacitors or inductors. Non-linear loads, such as rectifiers, distort the current drawn from the system. In such cases, active or passive power factor correction may be used to counteract the distortion and raise the power factor. The devices for correction of the power factor may be at a central substation, spread out over a distribution system, or built into power-consuming equipment.

General case

The general expression for power factor is given by

$$\text{Power factor} = P/P_a$$

$$P_a = I_{rms} V_{rms}$$

Where P is the real power measured by an ideal watmeter, I_{rms} is the rms current measuredby anideal ammeter, and V_{rms} is the rms voltage measured by an ideal voltmeter. Apparent power, Pa is the product of the rms current and the rms voltage.

If the load is sourcing power back toward the generator, then and will be negative

Periodic waveforms

If the waveforms are periodic with the same period which is much shorter than the averaging time of the physical meters, then the power factor can be computed by the following

$$\text{power factor} = P/P_a$$
$$P_a = I_{rms}V_{rms}$$
$$P = \frac{1}{T}\int_{t'}^{t'+T} i(t)v(t)dt$$
$$I_{rms}^2 = \frac{1}{T}\int_{t'}^{t'+T} i(t)^2 dt$$
$$V_{rms}^2 = \frac{1}{T}\int_{t'}^{t'+T} v(t)^2 dt$$

Where i (t) is the instantaneous current, v (t) is the instantaneous voltage, t^ is an arbitrary starting time, and T is the period of the waveforms.

Linear time-invariant circuits

Linear time-invariant circuits (referred to simply as *linear circuits* for the rest of this article), for example, circuits consisting of combinations of resistors, inductors, and capacitors have a sinusoidal response to the sinusoidal line voltage.[1] A linear load does not change the shape of the input waveform but may change the relative timing (phase) between voltage and current, due to its inductance or capacitance.

In a purely resistive AC circuit, voltage and current waveforms are in step (or in phase), changing polarity at the same instant in each cycle. All the power entering the load is consumed (or dissipated). Where reactive loads are present, such as with capacitors or inductors, energy storage in the loads results in a phase difference between the current and voltage waveforms. During each cycle of the AC voltage, extra energy, in addition to any energy consumed in the load, is temporarily stored in the load in electric or magnetic fields and then returned to the power grid a fraction of the period later. Electrical circuits containing predominantly resistive loads (incandescent lamps, heating elements) have a power factor of almost 1, but circuits containing inductive or capacitive loads (electric motors, solenoid valves, transformers, fluorescent lamp ballasts, and others) can have a power factor well below 1. In the electric power grid, reactive loads cause a continuous ebb and flow of nonproductive power. A circuit with a low power factor will use a greater amount of current to transfer a given quantity of real power than a circuit with a high power factor thus causing increased losses due to resistive heating in power lines, and requiring the use of higher-rated conductors and transformers (Figure 6.1).

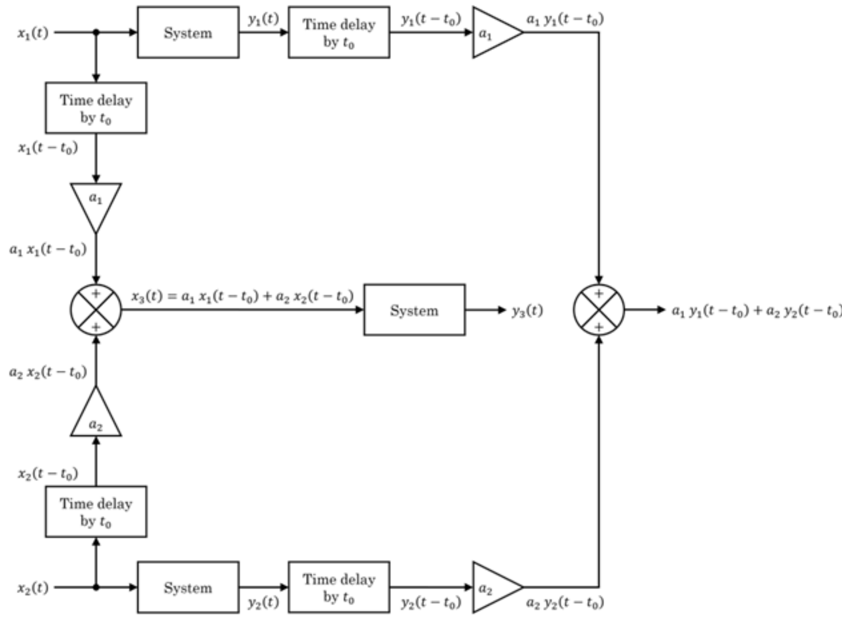

Figure 6.1: Linear time-invariant circuits

Importance of Power Factor

At very low power factor values, a large quantity of energy from the mains is wasted as a chunk of it will not be used for meaningful work due to the presence of more reactive loads indicated by the low power factor. This place a strain on the supply system as both the real power required by the load and the reactive power used to satisfy reactive loads will be drawn from the system to meet the requirements of the load. This strain and "wastage" typically leads to huge electricity bills for consumers(especially industrial consumers) as utility companies calculate consumption in terms of apparent power, as such, they end up paying for power that was not used to achieve any "meaningful" work. Even in situations where the power is being provided by the company's generators, money is wasted on bigger generators, larger-sized cables, etc., required to provide power when a good number of it is just going to waste.

How to calculate the power factor

To calculate the power factor, you need a power quality analyzer or power analyzer that measures both working power (kW) and apparent power (kVA) and calculates the ratio of kW/kVA. The power factor formula can be expressed in other ways[4]–[6]:

PF = (True power)/(Apparent power)

OR

PF = W/VA

Where wats measure useful power while VA measures supplied power. The ratio of the two is essentially useful power to supplied power, or:

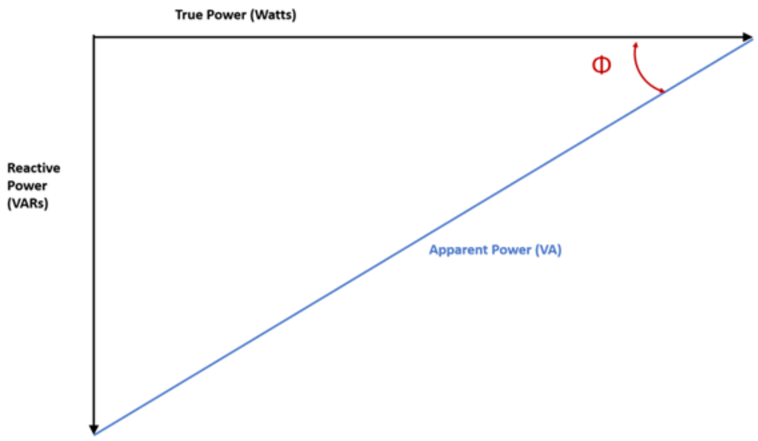

Figure 6.2: power factor compares the real power

As this Figure 6.2 demonstrates, the power factor compares the real power being consumed to the apparent power, or demand of the load. The power available to perform work is called real power. You can avoid power factor penalties by correcting for power factor. Poor power factor means that you're using power inefficiently. This maters to companies because it can result in:

1. Heat damage to insulation and other circuit components
2. Reduction in the amount of available useful power
3. A required increase in conductor and equipment sizes

Power factor correction of linear loads (Figure 6.3):

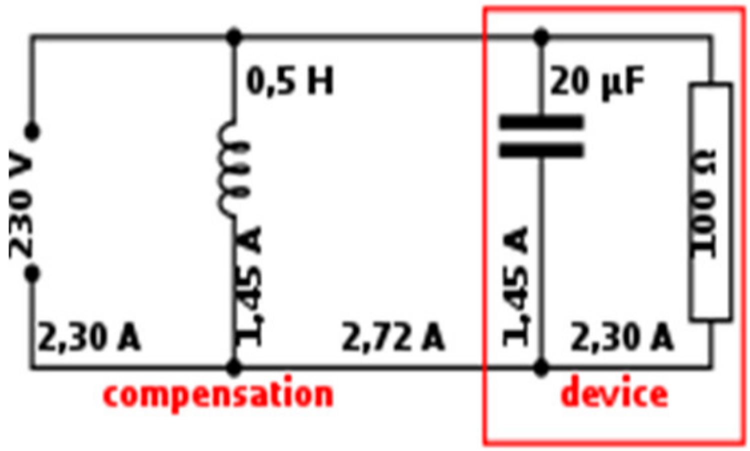

Figure 6.3: Power factor correction of linear load

A high power factor is generally desirable in a power delivery system to reduce losses and improve voltage regulation at the load. Compensating elements near an electrical load will reduce the apparent power demand on the supply system. Power factor correction may be applied by an electric power transmission utility to improve the stability and efficiency of the network. Individual electrical customers who are charged by their utility for low power factor may install correction equipment to increase their power factor to reduce costs.

Power factor correction brings the power factor of an AC power circuit closer to 1 by supplying or absorbing reactive power, adding capacitors or inductors that act to cancel the inductive or capacitive effects of the load, respectively. In the case of offseting the inductive effect of motor loads, capacitors can be locally connected. These capacitors help to generate reactive power to meet the demand of the inductive loads. This will keep that reactive power from having to flow from the utility generator to the load. In the electricity industry, inductors are said to consume reactive power, and capacitors are said to supply it, even though reactive power is just energy moving back and forth on each AC cycle.

The reactive elements in power factor correction devices can create voltage fluctuations and harmonic noise when switched on or off. They will supply or sink reactive power regardless of whether there is a corresponding load operating nearby, increasing the system's no-load losses. In the worst case, reactive elements can interact with the system and with each other to create resonant conditions, resulting in system instability and severe overvoltage fluctuations. As such, reactive elements cannot simply be applied without engineering analysis.

- Reactive power control relay;
- Network connection points;
- Slow-blow fuses;
- Inrush-limiting contractors;
- Capacitors (single-phase or three-phase units, delta-connection);
- Transformer (for controls and ventilation fans)

An automatic power factor correction unit consists of some capacitors that are switched using contactors. These contactors are controlled by a regulator that measures the power factor in an electrical network. Depending on the load and power factor of the network, the power factor controller will switch the necessary blocks of capacitors in steps to make sure the power factor stays above a selected value. In place of a set of switched capacitors, an unloaded synchronous motor can supply reactive power. The reactive power drawn by the synchronous motor is a function of its field excitation. It is referred to as a synchronous condenser. It is started and connected to the electrical network. It operates at a leading power factor and puts vars onto the network as required to support a system's voltage or to maintain the system power factor at a specified level.

The synchronous condenser's installation and operation are identical to those of large electric motors. Its principal advantage is the ease with which the amount of correction can be adjusted; it behaves like a variable capacitor. Unlike with capacitors, the amount of reactive power supplied is proportional to voltage, not the square of voltage; this improves voltage stability on large networks. Synchronous condensers are often used in connection with high-voltage direct-current transmission projects or large industrial plants such as steel mills.

For power factor correction of high-voltage power systems or large, fluctuating industrial loads, power electronic devices such as the static VAR compensator or STATCOM are increasingly used.

These systems can compensate for sudden changes in power factor much more rapidly than contactor-switched capacitor banks and, being solid-state, require less maintenance than synchronous condensers.

Non-linear load

Examples of non-linear loads on a power system are rectifiers (such as those used in a power supply), and arc discharge devices such as fluorescent lamps, electric welding machines, or arc furnaces. Because the current in these systems is interrupted by a switching action, the current contains frequency components that are multiples of the power system frequency. Distortion power factor is a measure of how much the harmonic distortion of a load current decreases the average power transferred to the load.

Non-sinusoidal components

In linear circuits having only sinusoidal currents and voltages of one frequency, the power factor arises only from the difference in phase between the current and voltage. This is the displacement power factor.

Non-linear loads change the shape of the current waveform from a sine wave to some other form. Non-linear loads create harmonic currents in addition to the original (fundamental frequency) AC. This is of importance in practical power systems that contain non-linear loads such as rectifiers, some forms of electric lighting, electric arc furnaces, welding equipment, switched-mode power supplies, variable speed drives, and other devices. Filters consisting of linear capacitors and inductors can prevent harmonic currents from entering the supplying system.

To measure the real power or reactive power, a watmeter designed to work properly with non-sinusoidal currents must be used.

Distortion in three-phase networks

In practice, the local effects of distortion current on devices in a three-phase distribution network rely on the magnitude of certain order harmonics rather than the total harmonic distortion.

For example, the triple, or zero-sequence, harmonics (3rd, 9th, 15th, etc.) have the property of being in a phase when compared line-to-line. In a delta-wye transformer, these harmonics can result in circulating currents in the delta windings and result in greater resistive heating. In a wye configuration of a transformer, triple harmonics will not create these currents, but they will result in a non-zero current in the neutral wire. This could overload the neutral wire in some cases and create an error in kilowat-hour metering systems and billing revenue The presence of current harmonics in a transformer also result in larger eddy currents in the magnetic core of the transformer. Eddy current losses generally increase as the square of the frequency, lowering the transformer's efficiency, dissipating additional heat, and reducing its service life[7]–[10]

Negative-sequence harmonics (5th, 11th, 17th, etc.) combine 120 degrees out of phase, similarly to the fundamental harmonic but in a reversed sequence. In generators and motors, these currents produce magnetic fields which oppose the rotation of the shaft and sometimes result in damaging mechanical vibrations

Switched-mode power supplies

Main article: switched-mode power supply § Power factor

A particularly important class of non-linear loads is the millions of personal computers that typically incorporate switched-mode power supplies (SMPS) with rated output power ranging from a few wats to more than 1 kW. Historically, these very-low-cost power supplies incorporated a simple full-wave rectifier that conducted only when the mains instantaneous voltage exceeded the voltage on the input capacitors. This leads to very high ratios of peak-to-average input current, which also leads to a low distortion power factor and potentially serious phase and neutral loading concerns.

A typical switched-mode power supply first converts the AC mains to a DC bus using a bridge rectifier. The output voltage is then derived from this DC bus. The problem with this is that the rectifier is a non-linear device, so the input current is highly non-linear. That means that the input current has energy at harmonics of the frequency of the voltage. This presents a problem for power companies because they cannot compensate for the harmonic current by adding simple capacitors or inductors, as they could for the reactive power drawn by a linear load. Many jurisdictions are beginning to require power factor correction for all power supplies above a certain power level.

Regulatory agencies such as the EU have set harmonic limits as a method of improving power factors. The declining component cost has hastened the implementation of two different methods. To comply with current EU standard EN61000-3-2, all switched-mode power supplies with output power more than 75 W must at least include passive power factor correction. 80 Plus power supply certification requires a power factor of 0.9 or more

Dynamic PFC

Dynamic power factor correction (DPFC), sometimes referred to as real-time power factor correction, is used for electrical stabilization in cases of rapid load changes (e.g. at large manufacturing sites). DPFC is useful when standard power factor correction would cause over or under-correction. DPFC uses semiconductor switches, typically thyristors, to quickly connect and disconnect capacitors or inductors to improve power factor.

Importance in distribution systems

Power factors below 1.0 require a utility to generate more than the minimum volt-amperes necessary to supply the real power (wats). This increases generation and transmission costs. For example, if the load power factor were as low as 0.7, the apparent power would be 1.4 times the real power used by the load. Line current in the circuit would also be 1.4 times the current required at 1.0 power factor, so the losses in the circuit would be doubled (since they are proportional to the square of the current). Alternatively, all components of the system such as generators, conductors, transformers, and switchgear would be increased in size (and cost) to carry the extra current. When the power factor is close to unity, for the same kVA rating of the transformer more load current can be supplied. Utilities typically charge additional costs to commercial customers who have a power factor below some limit, which is typically 0.9 to 0.95. Engineers are often interested in the power factor of a load as one of the factors that affect the efficiency of power transmission.

With the rising cost of energy and concerns over the efficient delivery of power, active PFC has become more common in consumer electronics.[30] Current Energy Star guidelines for computers[31] call for a power factor of ≥ 0.9 at 100% of the rated output in the PC's power supply. According to a white paper authored by Intel and the U.S. Environmental Protection

Agency, PCs with internal power supplies will require the use of active power factor correction to meet the ENERGY STAR 5.0 Program Requirements for Computers.

In Europe, EN 61000-3-2 requires power factor correction to be incorporated into consumer products. Small customers, such as households, are not usually charged for reactive power and so power factor metering equipment for such customers will not be installed.

Measurement techniques

The power factor in a single-phase circuit (or balanced three-phase circuit) can be measured with the watmeter-ammeter-voltmeter method, where the power in wats is divided by the product of measured voltage and current. The power factor of a balanced polyphase circuit is the same as that of any phase. The power factor of an unbalanced polyphase circuit is not uniquely defined. A direct reading power factor meter can be made with a moving coil meter of the electrodynamic type, carrying two perpendicular coils on the moving part of the instrument. The field of the instrument is energized by the circuit current flow. The two moving coils, A and B, are connected in parallel with the circuit load. One coil, A, will be connected through a resistor, and the second coil, B, through an inductor, so that the current in coil B is delayed concerning the current in A. At the unity power factor, the current in A is in phase with the circuit current, and coil A provides maximum torque, driving the instrument pointer toward the 1.0 mark on the scale. At zero power factor, the current in coil B is in phase with the circuit current, and coil B provides torque to drive the pointer toward 0. At intermediate values of power factor, the torques provided by the two coils add and the pointer takes up intermediate positions.

Another electromechanical instrument is the polarized-vane type. In this instrument, a stationary field coil produces a rotating magnetic field, just like a polyphase motor. The field coils are connected either directly to polyphase voltage sources or to a phase-shifting reactor if a single-phase application. A second stationary field coil, perpendicular to the voltage coils, carries a current proportional to the current in one phase of the circuit. The moving system of the instrument consists of two vanes that are magnetized by the current coil. In operation, the moving vanes take up a physical angle equivalent to the electrical angle between the voltage source and the current source. This type of instrument can be made to register for currents in both directions, giving a four-quadrant display of power factor or phase angle. Digital instruments exist that directly measure the time lag between voltage and current waveforms. Low-cost instruments of this type measure the peak of the waveforms. More sophisticated versions measure the peak of the fundamental harmonic only, thus giving a more accurate reading for phase angle on distorted waveforms. Calculating power factor from voltage and current phases is only accurate if both waveforms are sinusoidal.

Power Quality Analyzers, often referred to as Power Analyzers, make a digital recording of the voltage and current waveform (typically either one phase or three phases) and accurately calculate true power (wats), apparent power (VA) power factor, AC voltage, AC, DC voltage, DC, frequency, IEC61000-3-2/3-12 Harmonic measurement, IEC61000-3-3/3-11 flicker measurement, individual phase voltages in delta applications where there is no neutral line, total harmonic distortion, phase, and amplitude of the individual voltage or current harmonics, etc

In electrical engineering, the power factor of an AC power system is defined as the ratio of the real power absorbed by the load to the apparent power flowing in the circuit. Real power is the average of the instantaneous product of voltage and current and represents the capacity of the electricity for performing work. Apparent power is the product of RMS current and voltage. Due to energy stored in the load and returned to the source, or due to a non-linear load that distorts the wave shape of the current drawn from the source, the apparent power may be greater than the real power, so more current flows in the circuit than would be required to transfer real power alone. A power factor magnitude of less than one indicates the voltage and current are not in phase, reducing the average product of the two. A negative power factor occurs when the device (which is normally the load) generates real power, which then flows back toward the source.

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CHAPTER 7

ELECTRIC CIRCUITS AND SINGLE PHASE AC CIRCUITS

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In a dc circuit the relationship between the applied voltage V and the current flowing through circuit I is a simple one and is given by the look I = V/R but in an a-c circuit this simple association does not hold well. Differences in current and applied voltage set up magnetic and electrostatic effects singly and these must be taken into account with the resistance of the circuit while determining the quantitative relations between current and applied voltage. With reasonably low voltage, a heavy-current circuit's atractive effects may be very large, but electrostatic belongings are typically small. On the other hand with high-voltage circuits, electrostatic effects may be of appreciable magnitude, and magnetic effects are also present. Here it has been deliberated how the atractive effects due to differences in current do and electrostatic effects due to differences in the applied voltage affect the association between the applied voltage and current.

Purely Resistive Circuit

A purely resistive or a non-inductive circuit is a circuit that has inductance so minor that at usual frequency its reactance is insignificant as likened to its resistance. Normal thread spots, water resistances, etc., are examples of non-inductive confrontations. If the tour is purely non-inductive, no reactance emf (i.e., self-induced or back emf) is usually up and the completion of the applied voltage is applied in overwhelming the ohmic confrontation of the circuit. Reflect an ac circuit covering a non-inductive resistance of R ohms linked crossways a sinusoidal voltage signified by v = V sin wt, as shown in Figure 7.1:[1]–[3]

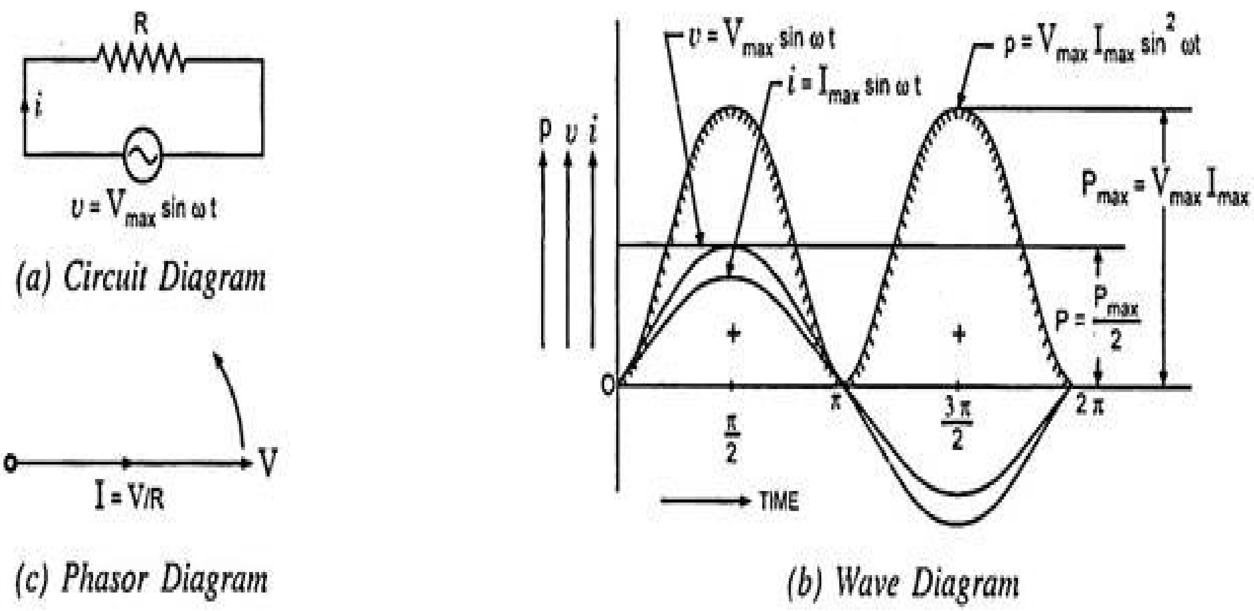


Figure 7.1: Purely Resistive Circuit

As already said, when the current flowing through pure resistance changes, no back emf is set up, therefore, applied voltage has to overcome the ohmic drop of I R only:

$$i.e. \quad iR = v$$

$$\text{or} \quad i = \frac{v}{R} = \frac{V_{max}}{R} \sin \omega t$$

Current will be maximum when $\omega t = \frac{\pi}{2}$ or $\sin \omega t = 1$

$$\therefore \quad I_{max} = \frac{V_{max}}{R}$$

$i = I_{max} \sin \omega t$

From the expressions of instantaneous applied voltage and instantaneous current, it is evident that in a purely resistive circuit, the applied voltage and current are in phase with each other, as shown by wave and phasor diagrams in Figure 7.1.

Power in Purely Resistive Circuit:

The instantaneous power delivered to the circuit in question is the product of the instantaneous values of applied voltage and current.

$$i.e. \quad p = vi = V_{max} \sin \omega t \, I_{max} \sin \omega t = V_{max} \; I_{max} \sin^2 \omega t$$

$$\text{or} \quad p = \frac{V_{max} \, I_{max}}{2} (1 - \cos 2\,\omega t) \qquad \text{Since } \sin^2 \omega t = \frac{1 - \cos 2\,\omega t}{2}$$

$$= \frac{V_{max} \, I_{max}}{2} - \frac{V_{max} \, I_{max}}{2} \cos 2\,\omega t$$

$$\text{Average power, } P = \text{Average of } \frac{V_{max} \, I_{max}}{2} - \textit{average of } \frac{V_{max} \, I_{max}}{2} \cos 2\,\omega t$$

Since average of $\frac{V_{max} \, I_{max}}{2} \cos 2\,\omega t$ over a complete cycle is zero,

$$P = \frac{V_{max} \, I_{max}}{2} = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = V\,I \text{ watts}$$

Where V and I are the rms values of applied voltage and current respectively.

Thus for purely resistive circuits, the expression for power is the same as for dc circuits. From the power curve for a purely resistive circuit, it is evident that power consumed in a purely resistive circuit is not constant, it is fluctuating. However, it is always positive. This is so because the instantaneous values of voltage and current are always either positive or negative and, therefore, the product is always positive. This means that the voltage source constantly delivers power to the circuit and the circuit consumes it.

Purely Inductive Circuit:

An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be had as the inductive coil has always small resistance. However, a coil of thick copper wire wound on a laminated iron core that has negligible resistance arid is known as a choke coil.

When an alternating voltage is applied to a purely inductive coil, an emf, known as self-induced emf, is induced in the coil which opposes the applied voltage. Since the coil has no resistance, at every instant applied voltage has to overcome this self-induced emf only.

Let the applied voltage $v = V_{max} \sin \omega t$

and self inductance of coil = L henry

Self induced emf in the coil, $e_L = -L \dfrac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self induced emf *i.e.* $v = -e_L$

$$\therefore \quad V_{max} \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

$$\text{or} \quad di = \frac{V_{max}}{L} \sin \omega t \, dt$$

Integrating both sides we get

$$i = \frac{V_{max}}{L} \int \sin \omega t \, dt = \frac{V_{max}}{\omega L} (-\cos \omega t) + A$$

where A is a constant of integration, which is found to be zero from initial conditions

$$i.e. \quad i = \frac{-V_{max}}{\omega L} \cos \omega t = \frac{V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Current will be maximum when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$, hence, maximum value of current,

$$I_{max} = \frac{V_{max}}{\omega L}$$

and instantaneous current may be expressed as $i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$

From the expressions of instantaneous applied voltage and the instantaneous current flowing through a purely inductive coil, it is observed that the current lags behind the applied voltage by π/2 by wave diagram and in Figure 7.2 by phasor diagram.

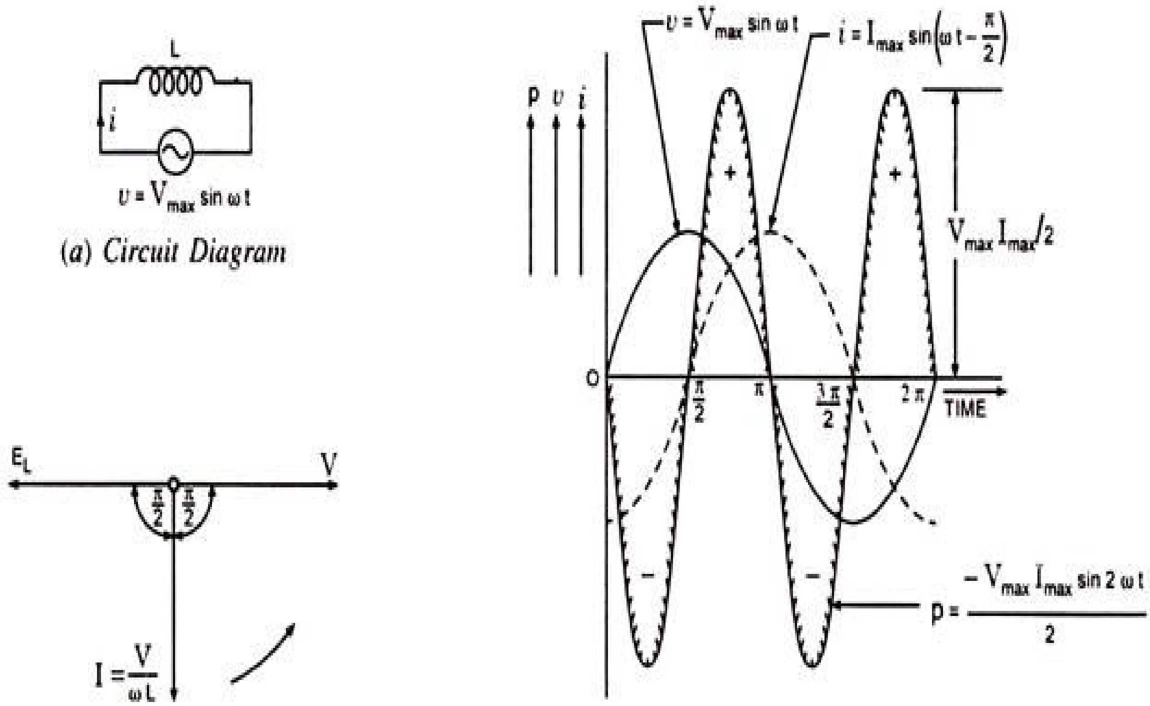


Figure 7.2: Purely Inductive Resistance Wave Diagram

Inductive Reactance:

ωL in the expression Imax = $V_{max}/\omega L$ is known as inductive reactance and is denoted by X_L i.e., $X_L = \omega L$

If L is in henry and co is in radians per second then X_L will be in ohms.

Power in Purely Inductive Circuit:

Instantaneous power, $p = v \times i = V_{max} \sin \omega t\ I_{max} \sin (\omega t - \pi/2)$

Or $p = - V_{max} I_{max} \sin \omega t \cos \omega t = V_{max} I_{max}/2 \sin 2 \omega t$

The power measured by the watmeter is the average value of p which is zero since the average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence in a purely inductive circuit power absorbed is zero.

Physically the above fact can be explained below:

During the second quarter of a cycle, the current and the magnetic flux of the coil increase and the coil draws power from the supply source to build up the magnetic field (the power drawn is positive and the energy drawn by the coil from the supply source is represented by the area between the curve p and the time axis). The energy stored in the magnetic field during build-up is given as $W_{max} = 1/2\ L\ I^2_{max}$.[4]–[6]

In the next quarter the current decreases. The emf of self-induction will, however, tends to oppose its decrease. The coil acts as a generator of electrical energy, returning the stored energy in the magnetic field to the supply source (now the power drawn by the coil is negative and the curve p lies below the time axis). The chain of events repeats itself during the next half cycles. Thus, a proportion of power is continually exchanged between the field and the inductive circuit and the power consumed by a purely inductive coil is zero.

Purely Capacitive Circuit:

When a dc voltage is impressed across the plates of a perfect condenser, it will become charged to full voltage almost instantaneously. The charging current will flow only during the period of "build-up" and will cease to flow as soon as the capacitor has atained the steady voltage of the source. This implies that for a direct current, a capacitor is a break in the circuit or an infinitely high resistance.

In a sinusoidal voltage is applied to a capacitor. During the first quarter cycle, the applied voltage increases to the peak value, and the capacitor is charged to that value. The current is maximum at the beginning of the cycle and becomes zero at the maximum value of the applied voltage, so there is a phase difference of 90° between the applied voltage and current. During the first quarter cycle the current flows in the normal direction through the circuit; hence the current is positive. In the second quarter-cycle, the voltage applied across the capacitor falls, the capacitor loses its charge, and current flows through it against the applied voltage because the capacitor discharges into the circuit. Thus, the current is negative during the second quarter cycle and atains a maximum value when the applied voltage is zero. The voltage applied across the capacitor falls the capacitor loses its change and the current flows (Figure 7.3).

The current is positive

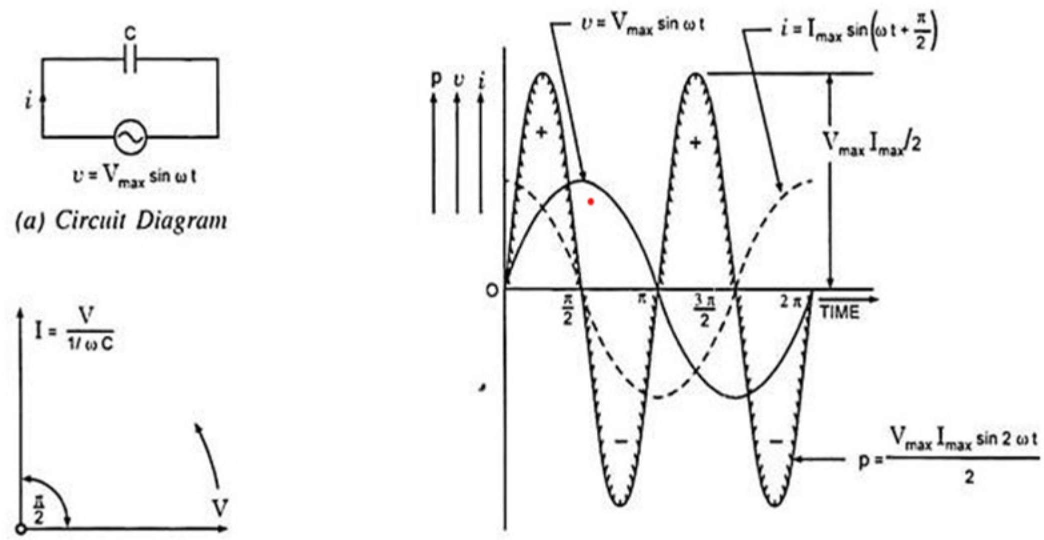


Figure 7.3: Purely Capacitive Circuit

The third and fourth quarter cycles repeat the events of the first and second, respectively, with the difference that the polarity of the applied voltage is reversed, and there are corresponding current changes. In other words, an alternating current flows in the circuit because of the charging and discharging of the capacitor. As illustrated in Figs and (c) the current begins its cycle 90 degrees ahead of the voltage, so the current in a capacitor leads the applied voltage by

90 degrees – the opposite of the inductance current-voltage relationship. Let an alternating voltage represented by $v = V_{max} \sin \omega t$ be applied across a capacitor of capacitance C farads.

The expression for the instantaneous charge is given as:

$q = C V_{max} \sin \omega t$

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation:

$$i = \frac{dq}{dt} = [C V_{max} \sin \omega t] = \omega C V_{max} \cos \omega t = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Current is maximum when $t = 0$

$$\therefore \quad I_{max} = \frac{V_{max}}{1/\omega C}$$

Substituting $\frac{V_{max}}{1/\omega C} = I_{max}$ in the above equation for instantaneous current, we get

$$i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the equations of instantaneous applied voltage and the instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by $\pi/2$, as shown in Figs. 4.4 (b) and (c) by wave and phasor diagrams respectively.

Capacitive Reactance:

$1/\omega C$ in the expression $I_{max} = V_{max}/1/\omega C$ is known as capacitive reactance and is denoted by X_C i.e., $XC = 1/\omega C$

If C is in farads and ω is in radians/s, then X_c will be in ohms.

Power in Purely Capacitive Circuit:

$$p = v\,i = V_{max} \sin \omega t . I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) = V_{max} I_{max} \sin \omega t \cos \omega t$$

$$= \frac{V_{max} I_{max}}{2} \sin 2\omega t$$

Average power, $P = \frac{V_{max} I_{max}}{2} \times$ average of $\sin 2\omega t$ over a complete cycle = 0.

Hence power absorbed in a purely capacitive circuit is zero. The energy taken from the supply circuit is stored in the capacitor during the first quarter cycle and returned during the next. The energy stored by a capacitor at the maximum voltage across its plates is given by the expression:

$$W_C = \frac{1}{2} C V^2_{max}$$

This can be realized when it is recalled that no heat is produced and no work is done while current is flowing through a capacitor. As a mater of fact, in commercial capacitors, there is a slight energy loss in the dielectric in addition to a minute I^2 R loss due to the flow of current over the plates having definite ohmic resistance.

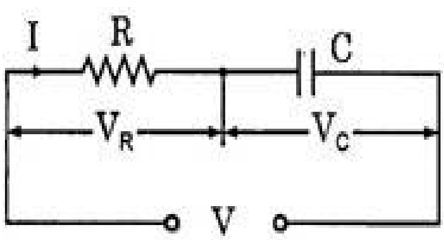
The power curve is a sine wave of double the supply frequency. Although it raises the power factor from zero to 0.002 or even a litle more, for ordinary purposes the power factor is taken to be zero. The phase angle due to dielectric and ohmic losses decreases slightly.

Resistance — Capacitance (R-C) Series Circuit:

Consider an ac circuit consisting of resistance of R ohms and capacitance of C farads connected in series, as shown in Fig. below that

Let the supply frequency be of *f* Hz and the current flowing through the circuit be I amperes (rms value). The voltage drop across resistance, $V_R = I R$ in phase with the current.

The voltage drop across capacitance, $V_C = I X_C$ lags behind me by π/2 radians or 90°, as shown in Figure 7.4.



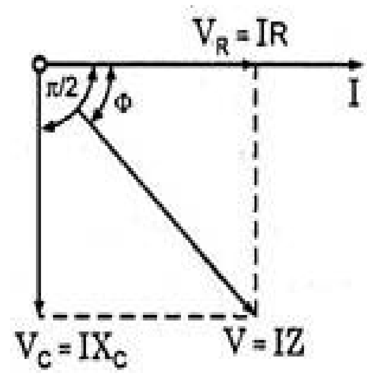


Figure 7.4: Resistance Capacitance (R-C) Series Circuit Diagram and Phasor Diagram

The applied voltage, being equal to the phasor sum of V_R and V_C, is given in magnitude by-

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{R^2 + X_C^2} = IZ$$

$$\text{where } Z^2 = R^2 + X_C^2$$

The applied voltage lags behind the current by an angle □:

where $\tan \Phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{\omega RC}$ or $\Phi = \text{Tan}^{-1} \frac{1}{R\omega C}$

Power factor, $\cos \Phi = \frac{R}{Z}$

If instantaneous voltage is represented by:

$v = V_{max} \sin \omega t$

Then the instantaneous current will be expressed as:

$i = I_{max}$ sin (ω t + □)

And power consumed by the circuit is given by:

P = VI cos □

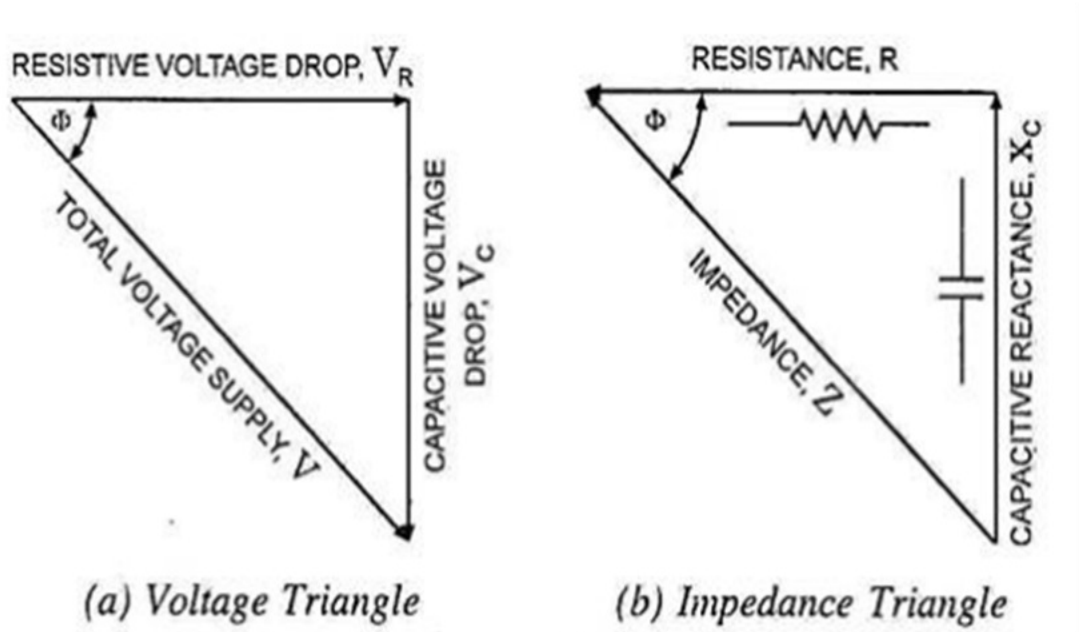


Figure 7.5: voltage triangle and impedance triangle

The voltage triangle and impedance triangle Figure 7.5

Apparent Power, True Power, Reactive Power, and Power Factor:

The product of rms values of current and voltage, VI is called the apparent power and is measured in volt-amperes or kilo-volt amperes (kVA). The true power in an ac circuit is obtained by multiplying the apparent power by the power factor and is expressed in wats or kilowats (kW). The product of apparent power, VI, and the sine of the angle between voltage and current, sin □ is called reactive power. This is also known as watless power and is expressed in reactive volt-amperes or kilo-volt amperes reactive (kVA R).[7]–[10]

$$i.e.\ \text{Apparent power, } S = VI \text{ volt-amperes or } \frac{VI}{1{,}000} \text{ kVA}$$

$$\text{True power, } P = VI\cos\Phi \text{ watts or } \frac{VI\cos\Phi}{1{,}000} \text{ kW}$$

$$\text{Reactive power, } Q = VI\sin\Phi \text{ VAR or } \frac{VI\sin\Phi}{1{,}000} \text{ kVAR}$$

$$\text{and kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2}$$

The above relations can easily be followed by referring to the power diagram shown in Figure 7.6.

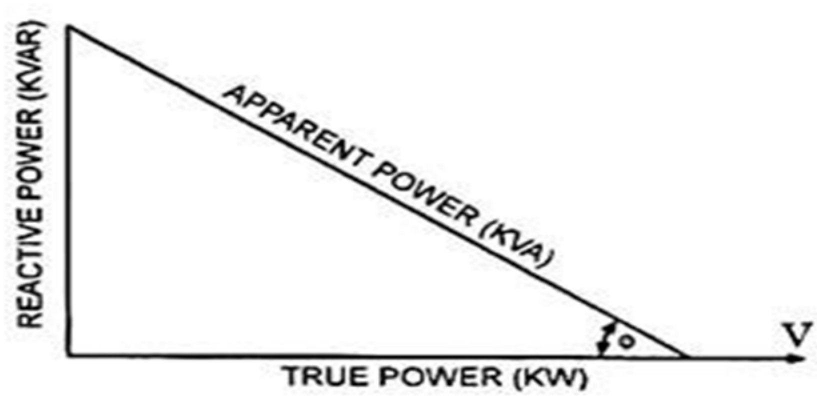


Figure 7.6: power diagram

Power factor may be defined as:

(i) Cosine of the phase angle between voltage and current

(ii) The ratio of the resistance to impedance, or

(iii) The ratio of true power to apparent power

The power factor can never be greater than unity. The power factor is expressed either as a fraction or as a percentage. It is usual practice to atach the word 'lagging' or ' leading' with the numerical value of the power factor to signify whether the current lags behind or leads the voltage.

Active Component of Current:

The current component which is in phase with circuit voltage (i.e., I cos □) and contributes to the active or true power of the circuit is called the active (watchful or in-phase) component of current

Reactive Component of Current:

The current component which is in quadrature (or 90° out of phase) to circuit voltage (i.e., I sin □) and contributes to the reactive power of the circuit, is called the reactive (or watless) component of current.

Conclusion

In a dc circuit the relationship between the applied voltage V and the current flowing through circuit I is a simple one and is given by the expression I = V/R but in an a-c circuit this simple relationship does not hold well. Variations in current and applied voltage set up magnetic and electrostatic effects respectively and these must be taken into account with the resistance of the circuit while determining the quantitative relations between current and applied voltage. With comparatively low voltage, a heavy-current circuit's magnetic effects may be very large, but electrostatic effects are usually negligible. On the other hand with high-voltage circuits, electrostatic effects may be of appreciable magnitude, and magnetic effects are also present.

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CHAPTER 8

KIRCHHOFF'S LAWS AND OHM'S LAWS

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To must a correct kind of electric circuit, it is vital to study the rules of electricity. One of the chief rules for resolving electronic circuits is Kirchhoff's Law. With the help of Kirchhoff's law, it converts very easily to solve the circuits fast and easily. So, Kirchhoff has given two laws, Kirchhoff's current law and Kirchhoff's voltage law. Kirchhoff's laws help to examine the circuit. Now, the question rises who Kirchhoff was? Gustav Robert Kirchhoff was a German physicist born on 12 March 1824, in Konigsberg, Russia. Originally, he started his study on the transmission of electricity. This study led him to formulate the two laws of Closed Electric Circuits in 1845, i.e. Kirchhoff's current law and Kirchhoff's voltage law.

In 1845, Gustav Robert Kirchhoff, a German physicist, gave laws explaining the conservation of energy and current in an electric circuit. These laws help analyze and calculate the electrical impedance and resistance of a complex AC circuit. Let us now understand this concept in detail. There are two Kirchhoff's laws:

Kirchhoff's current law is Kirchhoff's first law or Kirchhoff's Junction rule. According to the Junction Rule, in an electric circuit, the total current in a junction is equal to the sum of currents outside the junction[1]–[3].

Kirchhoff's Voltage law is also known as Kirchhoff's Second law or Kirchhoff's loop rule. According to the Loop rule, the sum of voltages around a closed electric circuit is zero.

Kirchhoff's current law

Kirchhoff's current law states, 'The current flowing into a node or a junction must be equal to the current flowing out of it.'

In other words, it states that the algebraic sum of all currents in the given electric circuit is equal to zero.

Thus, this law indicates the conservation of charge. In physics, the charge is a conserved quantity, i.e. the amount of charge entering is equal to the amount of charge coming out of it.

Kirchhoff's voltage law

Kirchhoff's voltage law states that in any complete loop within an electric circuit, the sum of all voltages across components that provide electrical energy must be equal to the sum of all voltages across the other elements in the same loop.

In other words, the algebraic sum of all voltages in a loop is equal to 0. To get the proper result, it is essential to maintain the direction, either clockwise or anticlockwise. This law indicates the law of conservation of energy.

The work is done by the electrical charges or on the electrical charges due to the electrical forces inside the electrical component. The total work done by the charge carriers on the rest component is equal to the total work done on the charge carriers due to electrical forces. Thus, it means that the potential differences across the element are to be 0.

This law also called Kirchhoff's second law, or Kirchhoff's loop rule states the following:

The directed sum of the potential differences (voltages) around any closed loop is zero.

Similarly to Kirchhoff's current law, the voltage law can be stated as:

$$\sum_{k=1}^{n} V_k = 0$$

Here, n is the total number of voltages measured.

Derivation of Kirchhoff's voltage law

A similar derivation can be found in

Consider some arbitrary circuit. Approximate the circuit with lumped elements, so that (time-varying) magnetic fields are contained to each component and the field in the region exterior to the circuit is negligible. Based on this assumption, the Maxwell–Faraday equation reveals that

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}$$

in the exterior region. If each of the components has a finite volume, then the exterior region is simply connected, and thus the electric field is conservative in that region. Therefore, for any loop in the circuit, we find that

$$\sum_i V_i = -\sum_i \int_{\mathcal{P}_i} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = 0$$

Where P_i are paths around the exterior of each of the components, from one terminal to another.

Note that this derivation uses the following definition for the voltage rise from a to b:

$$V_{a \to b} = -\int_{\mathcal{P}_{a \to b}} \mathbf{E} \cdot \mathrm{d}\mathbf{l}$$

However, the electric potential (and thus voltage) can be defined in other ways, such as via the Helmholtz decomposition.

Procedure to solve the problem:

The algebraic sum of voltages near a closed loop should be zero.

Draw the current direction and label the voltage direction. Remember that voltage on a voltage source is always positive to the negative end.

Define either clockwise or anticlockwise direction as voltage drop direction. Once the direction is defined, the same convention is used in every loop—the sign + for the voltage in the current direction and – otherwise

KIRCHHOFF'S Laws' Applications

This law analyzes how the current and voltage sources work in the electric circuit.

Applications in daily life:

1. In the deserts, days are very hot as sand is rough; therefore, it is a good heat absorber. Now by Kirchhoff's Laws, a Good absorber is a good emiter. So accordingly, the nights will be cool. That's why in deserts, days are hot and nights are cold.
2. This law is used to calculate the unknown values of current and voltages in the circuit.
3. Kirchhoff's law was the first law that helped the analysis and calculation of complex circuits become manageable and easy.
4. The Wheatstone bridge is an essential application of Kirchhoff's laws. It is also used in mesh and node analysis.

Modeling real circuits with lumped elements

The lumped element approximation for a circuit is accurate at low frequencies. At higher frequencies, leaked fluxes and varying charge densities in conductors become significant. To an extent, it is possible to still model such circuits using parasitic components. If frequencies are too high, it may be more appropriate to simulate the fields directly using finite element modeling or other techniques.

To model circuits so that both laws can still be used, it is important to understand the distinction between physical circuit elements and ideal lumped elements. For example, a wire is not an ideal conductor. Unlike an ideal conductor, wires can inductively and capacitive couple to each other (and to themselves), and have a finite propagation delay. Real conductors can be modeled in terms of lumped elements by considering parasitic capacitances distributed between the conductors to model capacitive coupling or parasitic (mutual) inductances to model inductive coupling.[4] Wires also have some self-inductance.

Limitations

Kirchhoff's circuit laws are the result of the lumped-element model and both depend on the model applying to the circuit in question. When the model is not applicable, the laws do not apply. The current law is dependent on the assumption that the net charge in any wire, junction, or lumped component is constant. Whenever the electric field between parts of the circuit is non-negligible, such as when two wires are capacitively coupled, this may not be the case. This occurs in high-frequency AC circuits, where the lumped element model is no longer applicable. For example, in a transmission line, the charge density in the conductor may be constantly changing.

On the other hand, the voltage law relies on the fact that the action of time-varying magnetic fields is confined to individual components, such as inductors. In reality, the induced electric field produced by an inductor is not confined, but the leaked fields are often negligible

Modeling real circuits with lumped elements

The lumped element approximation for a circuit is accurate at low frequencies. At higher frequencies, leaked fluxes and varying charge densities in conductors become significant. To an extent, it is possible to still model such circuits using parasitic components. If frequencies are too high, it may be more appropriate to simulate the fields directly using finite element modeling or other techniques. To model circuits so that both laws can still be used, it is important to understand the distinction between physical circuit elements and ideal lumped elements. For example, a wire is not an ideal conductor. Unlike an ideal conductor, wires can inductively and capacitive couple to each other (and to themselves), and have a finite propagation delay. Real conductors can be modeled in terms of lumped elements by considering parasitic capacitances distributed between the conductors to model capacitive coupling or parasitic (mutual) inductances to model inductive coupling. Wires also have some self-inductance

OHM'S LAW

Ohm’s law is the basic rule of electricity explains the relationship between electric current, voltage, and resistance. Ohm’s law was named after the German physicist Georg Ohm who discovered this rule or law[4]–[6].

Ohm’s law statement

Ohm’s law states that the electric current flowing through a conductor is directly proportional to the voltage and inversely proportional to the resistance. In other words, the electric current flowing through a conductor increases with an increase in voltage (If resistance is not changed) whereas the electric current flowing through a conductor decreases with an increase in resistance of the conductor (If voltage is not changed).

Ohm’s law is mathematically writen as

$$I=\frac{V}{R}$$

Where V = Voltage applied to a conductor,
I = Electric current flowing through the conductor,
R = Resistance of the conductor

Electric current, voltage, and resistance definition

Electric current: The number of free electrons that flows through a conductor in one second is called electric current. Electric current is measured in amperes (A). Voltage: The difference in electric potential energy of charged particles between two points within the electric field is called voltage. The free electrons at the higher potential have more electrical potential energy whereas the free electrons at the lower potential have less electrical potential energy. Voltage is measured in volts (V).

Resistance: Resistance is the opposite force that resists the flow of electrons. The electrons that are moving freely through the conductor will collide continuously with the atoms (which act as barriers). This causes the free electrons to lose their energy. Hence, the electric current decreases. Resistance is measured in ohm's (Ω).

Ohm's law explanation with an example

The concept of ohm's law is easily understood with the water analogy. The difference in water pressure between two points in a tank causes the water to flow. Here, the difference in the water pressure is compared with the voltage or potential difference. The rate at which water flows per second is compared with electric current. The obstacle that decreases the water flow is compared with resistance.

The voltage applied to a conductor

When the voltage is applied to the conductor, the free electrons gain kinetic energy and start flowing from the higher potential of the conductor to the lower potential of the conductor. During the journey from one end to the other end of the conductor the free electrons collide with the atoms or ions. When the free electrons moving in the conductor collides with the atoms, they lose their kinetic energy. The energy loss of free electrons is released in the form of heat. However, due to the continuous supply of external electric field or voltage, the free electrons again accelerate. The free electrons moving through the conductor again collide with atoms and lose their kinetic energy.

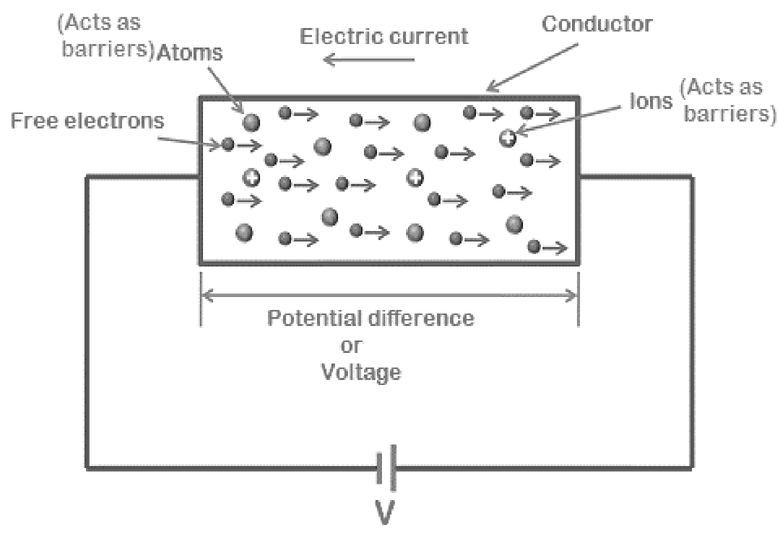

Figure 8.1: The voltage applied to a conductor

In this manner, the free electrons moving through the conductor collide continuously with atoms. Because of this continuous collision, the average velocity of free electrons decreases. Hence, the number of free electrons flowing from one point to another point within the conductor gets decreases. Therefore, the electric current decreases.

If the voltage applied to the conductor is increased

If the voltage or electric field applied to the conductor is increased, the free electrons gain a large amount of kinetic energy. Hence, the velocity of the free electrons increases. When the free electrons moving through the conductor collides with the atoms, they lose their kinetic energy. However, due to the continuous supply of voltage, the free electrons again atain their speed. The free electrons moving through the conductor again collide with the atoms and lose their kinetic energy. In this manner, the free electrons flowing through the conductor collide continuously with the atoms. Because of this continuous collision, the average velocity of the free electrons decreases. However, we provided more energy or electric field to the free electrons than in the previous case. Hence, the average velocity of the free electrons increases when compared with the previous case. Therefore, the electric current increases.

If the voltage applied to the conductor is held constant and resistance is increased

If the voltage or electric field applied to the free electrons in the conductor is held constant and increased the resistance of a conductor, the number of collisions with the atoms increases. As a result, the average drift velocity of the free electrons decreases. Therefore, the electric current decreases. The atractive force from the nucleus also affects the velocity of free electrons. The atractive force from the nucleus always tries to atract the free electrons. This decreases the velocity of free electrons. As a result, the electric current decreases.

Different arrangement of ohm's law equation

In electrical circuits, ohm's law equation is arranged in three different ways depending on what we want to solve.

Ohm's law equation for finding electric current:

To solve the electric current, ohm's law equation is writen as

The above equation tells us that, the electric current increases with an increase in voltage (If resistance is not changed) whereas the electric current decreases with an increase in resistance (If voltage is not changed).

Ohm's law equation for finding resistance:

To solve the resistance, ohm's law equation is writen as

The above equation tells us that, the resistance increases with an increase in voltage (If electric current is not changed) whereas the resistance decreases with an increase in electric current (If voltage is not changed).

Ohm's law equation for finding voltage:

To solve the voltage, ohm's law equation is writen as

$$V = IR$$

The above equation tells us that, the voltage increases with the increase in electric current (If resistance is not changed) or the voltage increases with the increase in resistance (If electric current is not changed).

Resistive circuits

Resistors are circuit elements that impede the passage of electric charge in agreement with Ohm's law and are designed to have a specific resistance value *R*. In schematic diagrams, a resistor is shown as a long rectangle or zig-zag symbol. An element (resistor or conductor) that behaves according to Ohm's law over some operating range is referred to as an ohmic device (or an ohmic resistor) because Ohm's law and a single value for the resistance suffice to describe the behavior of the device over that range. Ohm's law holds for circuits containing only resistive elements (no capacitances or inductances) for all forms of driving voltage or current, regardless of whether the driving voltage or current is constant (DC) or time-varying such as AC. At any instant, in time Ohm's law is valid for such circuits. Resistors that are in series or parallel may be grouped into a single "equivalent resistance" to apply Ohm's law in analyzing the circuit.

Reactive circuits with time-varying signals

When reactive elements such as capacitors, inductors, or transmission lines are involved in a circuit to which AC or time-varying voltage or current is applied, the relationship between voltage and current becomes the solution to a differential equation, so Ohm's law (as defined above) does not directly apply since that form contains only resistances having value *R*, not complex impedances which may contain capacitance (*C*) or inductance (*L*). Equations for time-invariant AC circuits take the same form as Ohm's law. However, the variables are generalized to complex numbers and the current and voltage waveforms are complex exponentials. In this approach, a voltage or current waveform takes the form Ae^{st}, where t is time, s is a complex parameter, and *A* is a complex scalar. In any linear time-invariant system, all of the currents and voltages can be expressed with the same *s* parameter as the input to the system, allowing the time-varying complex exponential term to be canceled out and the system described algebraically in terms of the complex scalars in the current and voltage waveforms[7]–[10].

The complex generalization of resistance is impedance, usually denoted *Z*; it can be shown that for an inductor,

$$Z = sL$$

and for a capacitor,

$$Z = \frac{1}{sC}.$$

We can now write,

$$V = Z\,I$$

where *V* and *I* are the complex scalars in the voltage and current respectively and *Z* is the complex impedance.

This form of Ohm's law, with Z taking the place of R, generalizes the simpler form. When Z is complex, only the real part is responsible for dissipating heat.

In a general AC circuit, Z varies strongly with the frequency parameter s, and so also will the relationship between voltage and current.

Relation to heat conductions

Ohm's principle predicts the flow of electrical charge (i.e. current) in electrical conductors when subjected to the influence of voltage differences; Jean-Baptiste-Joseph Fourier's principle predicts the flow of heat in heat conductors when subjected to the influence of temperature differences.

The same equation describes both phenomena, the equation's variables taking on different meanings in the two cases. Specifically, solving a heat conduction (Fourier) problem with temperature (the driving "force") and flux of heat (the rate of flow of the driven "quantity", i.e. heat energy) variables also solves an analogous electrical conduction (Ohm) problem having electric potential (the driving "force") and electric current (the rate of flow of the driven "quantity", i.e. charge) variables.

The basis of Fourier's work was his clear conception and definition of thermal conductivity. He assumed that all else being the same, the flux of heat is strictly proportional to the gradient of temperature. Although undoubtedly true for small temperature gradients, strictly proportional behavior will be lost when real materials (e.g. ones having a thermal conductivity that is a function of temperature) are subjected to large temperature gradients.

A similar assumption is made in the statement of Ohm's law: other things being alike, the strength of the current at each point is proportional to the gradient of electric potential. The accuracy of the assumption that flow is proportional to the gradient is more readily tested, using modern measurement methods, for the electrical case than for the heat case.

Thus, Kirchhoff's law is a fundamental electrical law that helps solve and analyze the electric circuit quickly. Calculating the unknown current and voltage in an electric circuit becomes easier. Kirchhoff's first law is based on the conservation of charges, and Kirchhoff's second law is based on energy conservation. Gustav Robert Kirchhoff described it in 1845. Kirchhoff's first law is the junction rule or current law, and Kirchhoff's second law is the loop rule or voltage law. It is the significant and fundamental law of electricity. Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points. Introducing the constant of proportionality, the resistance, one arrives at the usual mathematical equation that describes this relationship

Ohm's law is the basic rule of electricity explains the relationship between electric current, voltage, and resistance. Ohm's law was named after the German physicist Georg Ohm who discovered this rule or law.

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CHAPTER 9

LAPLACE TRANSFORM

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The Laplace transform is called afterward mathematician and astronomer Pierre-Simon, marquis de Laplace, who rummage-sale a similar transformation in his work on chance theory. Laplace marked lengthily the use of generating functions in Essay philosophies sunless probabilities (1814), and the integral form of the Laplace transform evolved naturally as a result. Laplace's use of producing functions was similar to what is now known as the z-transform, and he gave litle atention to the continuous variable case which was discussed by Niles Henrik Abel. The theory was further developed in the 19th and early 20th centuries by Mathias Larch, Oliver Heaviside, and Thomas Bromwich. The current widespread use of the transform (mainly in engineering) came about during and soon after World War II, replacing the earlier Heaviside operational calculus. The advantages of the Laplace transform had been emphasized by Gustav Doetsch, to whom the name Laplace transform is due.

From 1744, Leonhard Euler investigated integrals of the form

$$z = \int X(x) e^{ax}\, dx \quad \text{and} \quad z = \int X(x) x^{A}\, dx$$

As solutions of differential equations, but did not pursue the mater very far.[12] Joseph Louis Lagrange was an admirer of Euler and, in his work on integrating probability density functions, investigated expressions of the form[1]–[3]

$$\int X(x) e^{-ax} a^{x}\, dx,$$

Which some modern historians have interpreted within the modern Laplace transform theory.

These types of integrals seem first to have atracted Laplace's atention in 1782 when he was following in the spirit of Euler in using the integrals themselves as solutions of equations. However, in 1785, Laplace took the critical step forward when, rather than simply looking for a solution in the form of an integral, he started to apply the transforms in the sense that

$$\int x^s \varphi(x)\, dx,$$

Akin to a Mellin transform, to transform the whole of a difference equation, to look for solutions of the transformed equation. He then went on to apply the Laplace transform in the same way and started to derive some of its properties, beginning to appreciate its potential power.

Laplace also recognized that Joseph Fourier's method of Fourier series for solving the diffusion equation could only apply to a limited region of space because those solutions were periodic. In 1809, Laplace applied his transform to find solutions that diffused indefinitely in space.

Formal definition

The Laplace transform of a function f(t), defined for all real numbers $t \geq 0$, is the function F(s), which is a unilateral transform defined by

$$F(s) = \int_0^\infty f(t)e^{-st}\, dt$$

where s is a complex frequency domain parameter

$$s = \sigma + i\omega,$$

with real numbers σ and ω.

An alternate notation for the Laplace transform is $\mathcal{L}\{f\}$ instead of F.

The meaning of the integral depends on the types of functions of interest. A necessary condition for the existence of the integral is that f must be locally integrable on [0, ∞). For locally integrable functions that decay at infinity or are of exponential type, the integral can be understood to be a (proper) Lebesgue integral. However, for many applications, it is necessary to regard it as a conditionally convergent improper integral ∞. Still more generally, the integral can be understood in a weak sense, and this is dealt with below: One can define the Laplace transform of a finite Borel measure μ by the Lebesgue integral

$$\mathcal{L}\{\mu\}(s) = \int_{[0,\infty)} e^{-st}\, d\mu(t).$$

An important special case is where μ is a probability measure, for example, the Dirac delta function. In operational calculus, the Laplace transform of a measure is often treated as though the measure came from a probability density function f. In that case, to avoid potential confusion, one often writes.

$$\mathcal{L}\{f\}(s) = \int_{0^-}^{\infty} f(t)e^{-st}\,dt,$$

where the lower limit of 0^- is a shorthand notation for

$$\lim_{\varepsilon \to 0^+} \int_{-\varepsilon}^{\infty}.$$

This limit emphasizes that any point mass located at 0 is entirely captured by the Laplace transform. Although with the Lebesgue integral, it is not necessary to take such a limit, it does appear more naturally in connection with the Laplace–Stieltjes transform.

Bilateral Laplace transform

When one says "the Laplace transform" without qualification, the unilateral or one-sided transform is usually intended. The Laplace transform can be alternatively defined as the bilateral Laplace transform, or two-sided Laplace transforms, by extending the limits of integration to be the entire real axis. If that is done, the common unilateral transform simply becomes a special case of the bilateral transform, where the definition of the function being transformed is multiplied by the Heaviside step function.

The bilateral Laplace transform F(s) is defined as follows:

$$F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$$

An alternate notation for the bilateral Laplace transform is B{f}, instead of F

Inverse Laplace transform

Two integrable functions have the same Laplace transform only if they differ on a set of Lebesgue measure zero. This means that, on the range of the transform, there is an inverse transform. In fact, besides integrable functions, the Laplace transform is a one-to-one mapping from one function space into another in many other function spaces as well, although there is usually no easy characterization of the range.

Typical function spaces in which this is true include the spaces of bounded continuous functions, the space $L^\infty(0, \infty)$, or more generally tempered distributions on $(0, \infty)$. The Laplace transform is also defined and injective for suitable spaces of tempered distributions. In these cases, the image of the Laplace transforms lives in a space of analytic functions in the region of convergence. The inverse Laplace transform is given by the following complex integral, which is known by various names (the Bromwich integral, the Fourier–Mellin integral, and Mellin's inverse formula)[4]–[6]:

$$f(t) = \mathcal{L}^{-1}\{F\}(t) = \frac{1}{2\pi i} \lim_{T\to\infty} \int_{\gamma - iT}^{\gamma + iT} e^{st} F(s)\,ds$$

where γ is a real number so that the contour path of integration is in the region of convergence of F(s). In most applications, the contour can be closed, allowing the use of the residue theorem. An alternative formula for the inverse Laplace transform is given by Post's inversion formula. The limit here is interpreted in the weak-* topology. In practice, it is typically more convenient to decompose a Laplace transform into known transforms of functions obtained from a table, and construct the inverse by inspection.

Probability theory

In pure and applied probability, the Laplace transform is defined as an expected value. If X is a random variable with probability density function f, then the Laplace transform of f is given by the expectation

$$\mathcal{L}\{f\}(s) = \mathrm{E}\left[e^{-sX}\right].$$

By convention, this is referred to as the Laplace transform of the random variable X itself. Here, replacing s with –t gives the moment-generating function of X. The Laplace transform has applications throughout probability theory, including first passage times of stochastic processes such as Markov chains, and renewal theory. Of particular use is the ability to recover the cumulative distribution function of a continuous random variable X, using the Laplace transform as follows:

$$F_X(x) = \mathcal{L}^{-1}\left\{\frac{1}{s}\mathrm{E}\left[e^{-sX}\right]\right\}(x) = \mathcal{L}^{-1}\left\{\frac{1}{s}\mathcal{L}\{f\}(s)\right\}(x).$$

Region of convergence

If f is a locally integrable function (or more generally a Borel measure locally of bounded variation), then the Laplace transform F(s) of f converges provided that the limit:

$$\lim_{R\to\infty}\int_0^R f(t)e^{-st}\,dt$$

The Laplace transform converges absolutely if the integral

$$\int_0^\infty \left|f(t)e^{-st}\right|\,dt$$

Exists as a proper Lebesgue integral. The Laplace transform is usually understood as conditionally convergent, meaning that it converges in the former but not in the later sense. The set of values for which F(s) converges is either of the form Re(s) > a or Re(s) ≥ a, where a is an extended real constant with –∞ ≤ a ≤ ∞ (a consequence of the dominated convergence theorem). The constant a is known as the abscissa of absolute convergence and depends on the growth behavior of f(t).[20] Analogously, the two-sided transform converges absolutely in a strip of the form a < Re(s) < b, and possibly including the lines Re(s) = a or Re(s) = b. The subset of values of s for which the Laplace transform converges is called the region of absolute convergence, or the domain of absolute convergence. In the two-sided case, it is sometimes called the strip of absolute convergence. The Laplace transform is analytic in the region of absolute convergence: this is a consequence of Fubini's theorem and Morera's theorem.

Similarly, the set of values for which F(s) converges (conditionally or absolutely) is known as the region of conditional convergence, or simply the region of convergence (ROC). If the Laplace transform converges (conditionally) at $s = s_0$, then it automatically converges for all s with $Re(s) > Re(s_0)$. Therefore, the region of convergence is a half-plane of the form Re(s) > a, possibly including some points of the boundary line Re(s) = a.

In the region of convergence $Re(s) > Re(s_0)$, the Laplace transform of f can be expressed by integrating parts as the integral:

$$F(s) = (s - s_0)\int_0^\infty e^{-(s-s_0)t}\beta(t)\,dt, \quad \beta(u) = \int_0^u e^{-s_0 t} f(t)\,dt.$$

That is, F(s) can effectively be expressed, in the region of convergence, as the convergent Laplace transform of some other function. In particular, it is analytic. There are several Paley–Wiener theorems concerning the relationship between the decay properties of f, and the properties of the Laplace transform within the region of convergence. In engineering applications, a function corresponding to a linear time-invariant (LTI) system is stable if every bounded input produces a bounded output. This is equivalent to the absolute convergence of the Laplace transform of the impulse response function in the region $Re(s) \geq 0$. As a result, LTI systems are stable, provided that the poles of the Laplace transform of the impulse response function have the negative real part. This ROC is used in knowing about the causality and stability of a system.

Properties and theorems

The Laplace transform has several properties that make it useful for analyzing linear dynamical systems. The most significant advantage is that differentiation becomes multiplication, and integration becomes division, by s (reminiscent of the way logarithms change multiplication to the addition of logarithms). Because of this property, the Laplace variable s is also known as the operator variable in the L domain: either the derivative operator or (for s^{-1}) integration operator. The transform turns integral equations and differential equations into polynomial equations, which are much easier to solve. Once solved, the use of the inverse Laplace transform reverts to the original domain.

Given the functions f(t) and g(t), and their respective Laplace transforms F(s) and G(s),

$$f(t) = \mathcal{L}^{-1}\{F\}(s),$$
$$g(t) = \mathcal{L}^{-1}\{G\}(s),$$

Relation to power series

The Laplace transform can be viewed as a continuous analog of a power series. If a(n) is a discrete function of a positive integer n, then the power series associated with a(n) is the series

$$\sum_{n=0}^{\infty} a(n)x^n$$

where x is a real variable (see Z-transform). Replacing summation over n with integration over t, a continuous version of the power series becomes.

$$\int_0^{\infty} f(t)x^t\,dt$$

where the discrete function a(n) is replaced by the continuous one f(t).

Changing the base of the power from x to e gives

$$\int_0^{\infty} f(t)\left(e^{\ln x}\right)^t\,dt$$

For this to converge for, say, all bounded functions f, it is necessary to require that $\ln x < 0$. Making the substitution $-s = \ln x$ gives just the Laplace transform:

$$\int_0^{\infty} f(t)e^{-st}\,dt$$

In other words, the Laplace transform is a continuous analog of a power series, in which the discrete parameter n is replaced by the continuous parameter t, and x is replaced by e^{-s}.

Relation to moments

The quantities

$$\mu_n = \int_0^{\infty} t^n f(t)\,dt$$

are the moments of the function f. If the first n moments of f converge absolutely, then by repeated differentiation under the integral

$$\mu_n = (-1)^n \frac{d^n}{ds^n} \mathrm{E}\left[e^{-sX}\right](0).$$

This is of special significance in probability theory, where the moments of a random variable X are given by the expectation values $\mu_n = E[x^n]$. Then, the relationship holds

Computation of the Laplace transform of a function's derivative

It is often convenient to use the differentiation property of the Laplace transform to find the transform of a function's derivative. This can be derived from the basic expression for a Laplace transform as follows:

$$\mathcal{L}\left\{f(t)\right\} = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$$

$$= \left[\frac{f(t)e^{-st}}{-s}\right]_{0^-}^{\infty} - \int_{0^-}^{\infty} \frac{e^{-st}}{-s} f'(t)\,dt \quad \text{(by parts)}$$

$$= \left[-\frac{f(0^-)}{-s}\right] + \frac{1}{s}\mathcal{L}\left\{f'(t)\right\},$$

yielding

$$\mathcal{L}\{f'(t)\} = s\cdot\mathcal{L}\{f(t)\} - f(0^-),$$

and in the bilateral case,

$$\mathcal{L}\{f'(t)\} = s\int_{-\infty}^{\infty} e^{-st} f(t)\,dt = s\cdot\mathcal{L}\{f(t)\}.$$

The general result

$$\mathcal{L}\left\{f^{(n)}(t)\right\} = s^n\cdot\mathcal{L}\{f(t)\} - s^{n-1}f(0^-) - \cdots - f^{(n-1)}(0^-),$$

where $f^{(n)}$ denotes the nth derivative of f, which can then be established with an inductive argument.

Fourier transform

Further information: Fourier transform § Laplace transform

The Fourier transform is a special case (under certain conditions) of the bilateral Laplace transform. While the Fourier transform of a function is a complex function of a real variable (frequency), the Laplace transform of a function is a complex function of a complex variable. The Laplace transform is usually restricted to the transformation of functions of t with $t \geq 0$. A consequence of this restriction is that the Laplace transform of a function is a holomorphic function of the variable s. Unlike the Fourier transforms, the Laplace transform of a distribution is generally a well-behaved function. Techniques of complex variables can also be used to directly study Laplace transforms. As a holomorphic function, the Laplace transform has a power series representation. This power series expresses a function as a linear superposition of moments of the function.

This convention of the Fourier transform (in Fourier transform § Other conventions) requires a factor of $1/2\pi$ on the inverse Fourier transform. This relationship between the Laplace and Fourier transforms is often used to determine the frequency spectrum of a signal or dynamical system. The above relation is valid as stated if and only if the region of convergence (ROC) of F(s) contains the imaginary axis, $\sigma = 0$.

Fundamental relationships

Since an ordinary Laplace transform can be writen as a special case of a two-sided transform, and since the two-sided transform can be writen as the sum of two one-sided transforms, the theory of the Laplace-, Fourier-, Mellin-, and Z-transforms are at botom the same subject. However, a different point of view and different characteristic problems are associated with each of these four major integral transforms[7]–[10].

Examples and applications

The Laplace transform is used frequently in engineering and physics; the output of a linear time-invariant system can be calculated by convolving its unit impulse response with the input signal. Performing this calculation in Laplace space turns the convolution into a multiplication; the later being easier to solve because of its algebraic form. For more information, see control theory. The Laplace transform is invertible on a large class of functions. Given a simple mathematical or functional description of an input or output to a system, the Laplace transform provides an alternative functional description that often simplifies the process of analyzing the behavior of the system, or synthesizing a new system based on a set of specifications.

The Laplace transform can also be used to solve differential equations and is used extensively in mechanical engineering and electrical engineering. The Laplace transform reduces a linear differential equation to an algebraic equation, which can then be solved by the formal rules of the algebra. The original differential equation can then be solved by applying the inverse Laplace transform. English electrical engineer Oliver Heaviside first proposed a similar scheme, although without using the Laplace transform; and the resulting operational calculus is credited as the Heaviside calculus.

S-domain equivalent circuits and impedances

The Laplace transform is often used in circuit analysis, and simple conversions to the s-domain of circuit elements can be made. Circuit elements can be transformed into impedances, very similar to phasor impedances.

Note that the resistor is the same in the time domain and the s-domain. The sources are put in if there are initial conditions on the circuit elements. For example, if a capacitor has an initial voltage across it, or if the inductor has an initial current through it, the sources inserted in the s-domain account for that. The equivalents for current and voltage sources are simply derived from the transformations in the table above.

The Laplace transform is called afterward mathematician and astronomer Pierre-Simon, marquis de Laplace, who rummage-sale a similar transformation in his work on chance theory. Laplace marked lengthily the use of generating functions in Essay philosophies sunless probabilities (1814), and the integral form of the Laplace transform evolved naturally as a result. Laplace's use of producing functions was similar to what is now known as the z-transform, and he gave litle atention to the continuous variable case which was discussed by Niles Henrik Abel. The theory was further developed in the 19th and early 20th centuries by Mathias Larch, Oliver Heaviside, and Thomas Bromwich. The current widespread use of the transform (mainly in engineering) came about during and soon after World War II, replacing the earlier Heaviside operational calculus. The advantages of the Laplace transform had been emphasized by Gustav Doetsch, to whom the name Laplace transform is due.

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CHAPTER 10

TUNED CIRCUITS AND LC CIRCUIT

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An LC circuit, also called a ringing circuit, tank circuit, or tuned circuit, is an electric circuit consisting of an inductor, represented by the dispatch L, and a capacitor, characterized by the leter C, connected composed. The circuit can act as an electrical resonator, an electrical similarity of a tuning fork, storage energy oscillating at the circuit's booming frequency. AN LC Circuits are a type of electric circuit that is made up of an inductor which is expressed by the leter L and a capacitor represented by the leter C. Here, both are connected in a single circuit. An LC circuit is also sometimes referred to as a tank circuit, resonant circuit, or tuned circuit. LC circuits act as major components in various electronic devices like radio equipment, in circuits like filters, oscillators, and tuners (Figure 10.1).

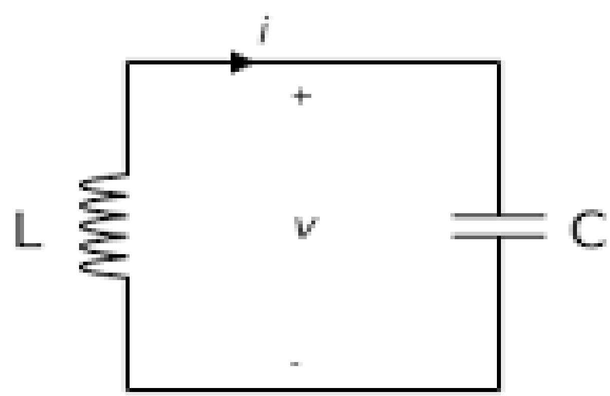


Figure 10.1: LC Circuit diagram

LC circuit is a suitable model in many cases mainly because by using this type of circuit we can assume that there is no dissipation of energy even if there is any resistance. However, if we were to be practical any implementation will involve loss because of the small electrical resistance in the connecting wires or components. This type of circuit is used because it can oscillate with minimum damping making the resistance as low as possible. Nonetheless, most of the circuits work with some loss. When an LC circuit oscillates at its natural resonant frequency, it can reserve electrical energy. The capacitor depending on the voltage it receives will store energy in the electric field (E) between its plates whereas an inductor depending on the current, will accumulate energy in its magnetic field[1]–[3]

Nomenclature

The di elemental LC circuit that we talked about in the above paragraphs is a basic example of an inductor-capacitor network. Moreover, it is also called a second-order LC circuit to differentiate it from highly complicated LC networks that have more capacitors and inductors. These LC networks that consist of more than two reactances can consist of several resonant frequencies. A resonant frequency is defined as an undamped or natural frequency of a system. In the case of LC circuits, the resonant frequency is usually determined by the impedance L and capacitance C.

Meanwhile, the network order is an order of rational functions which describes the network in complex frequency variables s. The order generally equals the number of L and C elements of the circuit and cannot exceed in any event.

Terminology

The two-element LC circuit described above is the simplest type of inductor-capacitor network (or LC network). It is also referred to as a second-order LC circuit to distinguish it from more complicated (higher-order) LC networks with more inductors and capacitors. Such LC networks with more than two reactances may have more than one resonant frequency. The order of the network is the order of the rational function describing the network in the complex frequency variable *s*. Generally, the order is equal to the number of L and C elements in the circuit and any event cannot exceed this number.

Operation

An LC circuit, oscillating at its natural resonant frequency, can store electrical energy. See the animation. A capacitor stores energy in the electric field (E) between its plates, depending on the voltage across it, and an inductor stores energy in its magnetic field (B), depending on the current through it. If an inductor is connected across a charged capacitor, the voltage across the capacitor will drive a current through the inductor, building up a magnetic field around it. The voltage across the capacitor falls to zero as the charge is used up by the current flow. At this point, the energy stored in the coil's magnetic field induces a voltage across the coil, because inductors oppose changes in current. This induced voltage causes a current to begin to recharge the capacitor with a voltage of opposite polarity to its original charge. Due to Faraday's law, the EMF which drives the current is caused by a decrease in the magnetic field, thus the energy required to charge the capacitor is extracted from the magnetic field. When the magnetic field is completely dissipated the current will stop and the charge will again be stored in the capacitor, with the opposite polarity as before. Then the cycle will begin again, with the current flowing in the opposite direction through the inductor.

The charge flows back and forth between the plates of the capacitor, through the inductor. The energy oscillates back and forth between the capacitor and the inductor until (if not replenished from an external circuit) internal resistance makes the oscillations die out. The tuned circuit's action, known mathematically as a harmonic oscillator, is similar to a pendulum swinging back and forth, or water sloshing back and forth in a tank; for this reason, the circuit is also called a *tank* circuit. The natural frequency (that is, the frequency at which it will oscillate when isolated from any other system, as described above) is determined by the capacitance and inductance values. In most applications the tuned circuit is part of a larger circuit that applies alternating current to it, driving continuous oscillations. If the frequency of the applied current is the circuit's natural resonant frequency (natural frequency, resonance will occur, and a small driving current can excite large amplitude oscillating voltages and currents. In typically tuned circuits in electronic equipment, It appears that you are missing a comma after the introductory phrase In typically tuned circuits in electronic equipment. Consider adding a comma the oscillations are very fast, from thousands to billions of times per second

Resonance

occurs when an LC circuit is driven from an external source at an angular frequency ω_0 at which the inductive and capacitive reactances are equal in magnitude. The frequency at which this equality holds for the particular circuit is called the resonant frequency. The resonant frequency of the LC circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

where L is the inductance in henries, and C is the capacitance in farads. The angular frequency ω_0 has units of radians per second.

The equivalent frequency in units of hertz is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

Application

The resonance effect of the LC circuit has many important applications in signal processing and communications systems.

1. The most common application of tank circuits is tuning radio transmiters and receivers. For example, when tuning a radio to a particular station, the LC circuits are set at resonance for that particular carrier frequency.
2. A series resonant circuit provides voltage magnification.
3. A parallel resonant circuit provides current magnification.
4. A parallel resonant circuit can be used as load impedance in output circuits of RF amplifiers. Due to high impedance, the gain of the amplifier is maximum at the resonant frequency.
5. Both parallel and series resonant circuits are used in induction heating.

LC circuits behave as electronic resonators, which are a key component in many applications:

1. Amplifiers
2. Oscillators
3. Filters
4. Tuners
5. Mixers

6. Foster–Seeley discriminator
7. Contactless cards
8. Graphics tablets
9. Electronic article surveillance (security tags)

Time domain solution

Kirchhoff's laws

By Kirchhoff's voltage law, the voltage V_C across the capacitor plus the voltage V_L across the inductor must equal zero:

$$V_C + V_L = 0.$$

Likewise, by Kirchhoff's current law, the current through the capacitor equals the current through the inductor:

$$I_C = I_L.$$

From the constitutive relations for the circuit elements, we also know that

$$V_L(t) = L\frac{\mathrm{d}I_L}{\mathrm{d}t},$$
$$I_C(t) = C\frac{\mathrm{d}V_C}{\mathrm{d}t}.$$

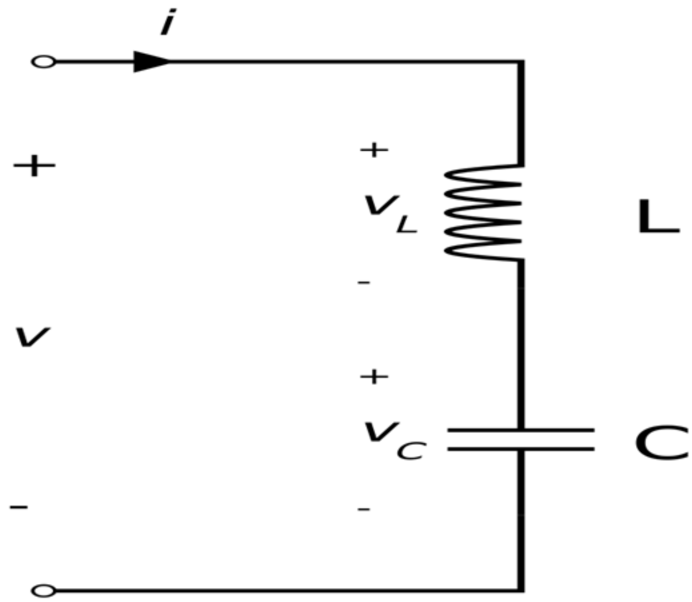
Series circuits

In the series configuration of the LC circuit, the inductor (L) and capacitor (C) are connected in series, as shown here.

The total voltage V across the open terminals is simply the sum of the voltage across the inductor and the voltage across the capacitor.

The current I into the positive terminal of the circuit is equal to the current through both the capacitor and the inductor (Figure 10.2).

$$V = V_L + V_C,$$
$$I = I_L = I_C.$$



Figuar 10.2: Series LC circuit

Resonance

Inductive reactance $X_L = wL$ increases as frequency increases, while capacitive reactance $X_C = \frac{1}{\omega C}$ decreases with an increase in frequency (defined here as a positive number). At one particular frequency, these two reactances are equal and the voltages across them are equal and opposite in sign; that frequency is called the resonant frequency f_0 for the given circuit

Hence, at resonance,

$$X_L = X_C,$$
$$\omega L = \frac{1}{\omega C}.$$

Solving for ω, we have

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}},$$

which is defined as the resonant angular frequency of the circuit. Converting angular frequency (in radians per second) into frequency (in hertz), one has

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

In a series configuration, X_C and X_L cancel each other out. In real, rather than idealized, components, the current is opposed, mostly by the resistance of the coil windings. Thus, the current supplied to a series resonant circuit is maximal at resonance.

1. In the limit as $f \rightarrow f_0$ current is maximal. Circuit impedance is minimal. In this state, a circuit is called an acceptor circuit
2. For $f < f_0$, $X_L \ll X_C$. Hence, the circuit is capacitive.
3. For $f > f_0$, $X_L \gg X_C$. Hence, the circuit is inductive.

Impedance

In the series configuration, resonance occurs when the complex electrical impedance of the circuit approaches zero. First, consider the impedance of the series LC circuit. The total impedance is given by the sum of the inductive and capacitive impedances[4]–[6]:

$$Z = Z_L + Z_C .$$

Writing the inductive impedance as $Z_L = j\omega L$ and capacitive impedance as $Z_C = 1/j\omega C$ and substituting gives

$$Z(\omega) = j\omega L + \frac{1}{j\omega C} .$$

Writing this expression under a common denominator gives

$$Z(\omega) = j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$

Finally, defining the natural angular frequency as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

the impedance becomes

$$Z(\omega) = j\,L\left(\frac{\omega^2 - \omega_0^2}{\omega}\right) = j\,\omega_0 L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

where w_0L gives the reactance of the inductor at resonance.

The numerator implies that in the limit as $\omega \rightarrow \pm\omega_0$, the total impedance Z will be zero and otherwise non-zero. Therefore the series LC circuit, when connected in series with a load, will act as a band-pass filter having zero impedance at the resonant frequency of the LC circuit.

Parallel circuit

When the inductor (L) and capacitor (C) are connected in parallel as shown here, the voltage V across the open terminals is equal to both the voltage across the inductor and the voltage across the capacitor. The total current I flowing into the positive terminal of the circuit is equal to the sum of the current flowing through the inductor and the current flowing through the capacitor (Figure 10.3):

$$V = V_L = V_C,$$
$$I = I_L + I_C.$$

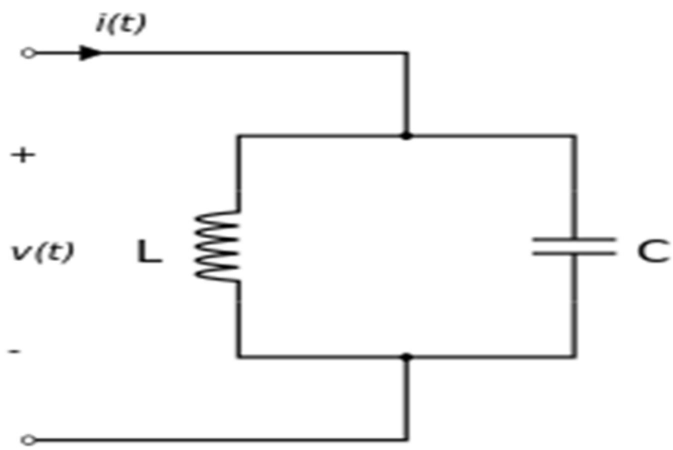


Figure:10.3: Parallel ac circuit

Resonance

When X_L equals X_C, the two branch currents are equal and opposite. They cancel out each other to give minimal current in the main line (in principle, zero current). However, there is a large current circulating between the capacitor and the inductor. In principle, this circulating current is infinite, but in reality is limited by resistance in the circuit, particularly resistance in the inductor windings. Since the total current is minimal, in this state the total impedance is maximal.

The resonant frequency is given by

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}.$$

Note that any branch current is not minimal at resonance, but each is given separately by dividing source voltage (V) by reactance (Z). Hence $I = V/Z$, as per Ohm's law.

1. At f_0, the line current is minimal. The total impedance is maximal. In this state, a circuit is called a rejector circuit.

2. Below f_0, the circuit is inductive.
3. Above f_0, the circuit is capacitive.

Impedance

The same analysis may be applied to the parallel LC circuit. The total impedance is then given by

$$Z = \frac{Z_L Z_C}{Z_L + Z_C},$$

and after substitution of $Z_L = j\omega L$ and $Z_C = 1/j\omega C$ and simplification, gives

$$Z(\omega) = -j \cdot \frac{\omega L}{\omega^2 LC - 1}.$$

Using

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

it further simplifies to

$$Z(\omega) = -j \left(\frac{1}{C}\right) \left(\frac{\omega}{\omega^2 - \omega_0^2}\right).$$

Note that

$$\lim_{\omega \to \omega_0} Z(\omega) = \infty,$$

but for all other values of ω the impedance is finite. Thus, the parallel LC circuit connected in series with a load will act as a band-stop filter having infinite impedance at the resonant frequency of the LC circuit, while the parallel LC circuit connected in parallel with a load will act as a band-pass filter.

History

The first evidence that a capacitor and inductor could produce electrical oscillations was discovered in 1826 by French scientist Felix Savary. He found that when a Leyden jar was discharged through a wire wound around an iron needle, sometimes the needle was left magnetized in one direction and sometimes in the opposite direction. He correctly deduced that this was caused by a damped oscillating discharge current in the wire, which reversed the magnetization of the needle back and forth until it was too small to have an effect, leaving the needle magnetized in a random direction. American physicist Joseph Henry repeated Savary's experiment in 1842 and came to the same conclusion, apparently independently.

Irish scientist William Thomson (Lord Kelvin) 1853 showed mathematically that the discharge of a Leyden jar through an inductance should be oscillatory, and derived from its resonant frequency. British radio researcher Oliver Lodge, by discharging a large batery of Leyden jars through a long wire, created a tuned circuit with its resonant frequency in the audio range, which produced a musical tone from the spark when it was discharged. In 1857, German physicist Berend Wilhelm Feddersen photographed the spark produced by a resonant Leyden jar circuit in a rotating mirror, providing visible evidence of the oscillations. In 1868, Scotish physicist James Clerk Maxwell calculated the effect of applying an alternating current to a circuit with inductance and capacitance, showing that the response is maximum at the resonant frequency. The first example of an electrical resonance curve was published in 1887 by German physicist Heinrich Hertz in his pioneering paper on the discovery of radio waves, showing the length of spark obtainable from his spark-gap LC resonator detectors as a function of frequency[7]–[10].

One of the first demonstrations of resonance between tuned circuits was Lodge's "syntonic jars" experiment around 1889. He placed two resonant circuits next to each other, each consisting of a Leyden jar connected to an adjustable one-turn coil with a spark gap. When a high voltage from an induction coil was applied to one tuned circuit, creating sparks and thus oscillating currents, sparks were excited in the other tuned circuit only when the circuits were adjusted to resonance. Lodge and some English scientists preferred the term "syntony" for this effect, but the term "resonance" eventually stuck.

The first practical use for LC circuits was in the 1890s in spark-gap radio transmiters to allow the receiver and transmiter to be tuned to the same frequency. The first patent for a radio system that allowed tuning was filed by Lodge in 1897, although the first practical systems were invented in 1900 by Italian radio pioneer Guglielmo Marconi.

LC TUNED CIRCUIT WORKING ANIMATION

diagram showing the operation of a tuned circuit (LC circuit). This is an improvement over previous versions of this animation which uses variable frame display times to show a more realistic movement of the charge. A tuned circuit is a very simple electronic circuit widely used in audio, radio, and television equipment.

It consists of an inductor *L* (coil of wire, left) and a capacitor *C* (right) connected. Tuned circuits can store electrical energy oscillating at its resonant frequency. The capacitor stores energy in its electric field *E* and the inductor stores energy in its magnetic field B. When the capacitor is charged with electricity and connected to the coil, the charge flows back and forth between the capacitor's plates as current I (red) through the coil as shown. This jerky animation shows "snapshots" of the circuit at progressive points in the oscillation. The oscillations are slowed down; in an actual tuned circuit, the charge oscillates back and forth hundreds of thousands to billions of times per second.

An LC circuit, also called a ringing circuit, tank circuit, or tuned circuit, is an electric circuit consisting of an inductor, represented by the dispatch L, and a capacitor, characterized by the leter C, connected composed. The circuit can act as an electrical resonator, an electrical similarity of a tuning fork, storage energy oscillating at the circuit's booming frequency. AN LC Circuits are a type of electric circuit that is made up of an inductor which is expressed by the leter L and a capacitor represented by the leter C. Here, both are connected in a single circuit. An LC circuit is also sometimes referred to as a tank circuit, resonant circuit, or tuned circuit. LC circuits act as major components in various electronic devices like radio equipment, in circuits like filters, oscillators, and tuners

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CHAPTER 11

COUPLED CIRCUIT

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Two circuits are said to be ‘coupled’ when energy transfer takes place from one circuit to the other when one of the circuits is energized. There are many types of Coupled Circuits conductive coupling as shown by the potential divider in Figure 11.1(a), inductive or magnetic coupling as shown by a two-winding transformer in Figure 11.1(b) or conductive and inductive coupling as shown by an autotransformer in Figure 11.1(c)

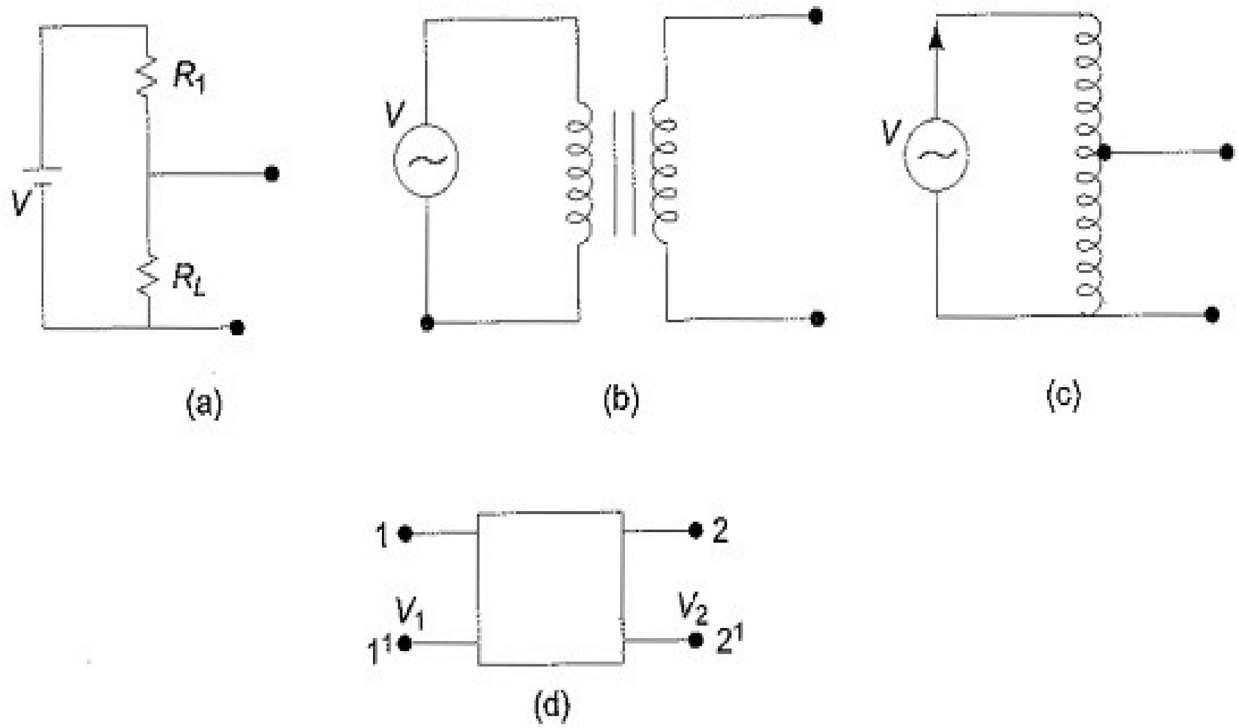

Figure 11.1:Coupled circuit

circuits. Each of these elements may be represented as a two-port network as shown in Figure 11.1(d). A majority of the electrical circuits in practice are conductively or electromagnetically coupled. Certain coupled elements are frequently used in network analysis and synthesis. Transformers, transistors, electronic pots, etc. are some of these[1]–[3]

Conductivity Coupled Circuit and Mutual Impedance:

A conductively Coupled Circuit that does not involve magnetic coupling is shown in Figure 11.2(a).

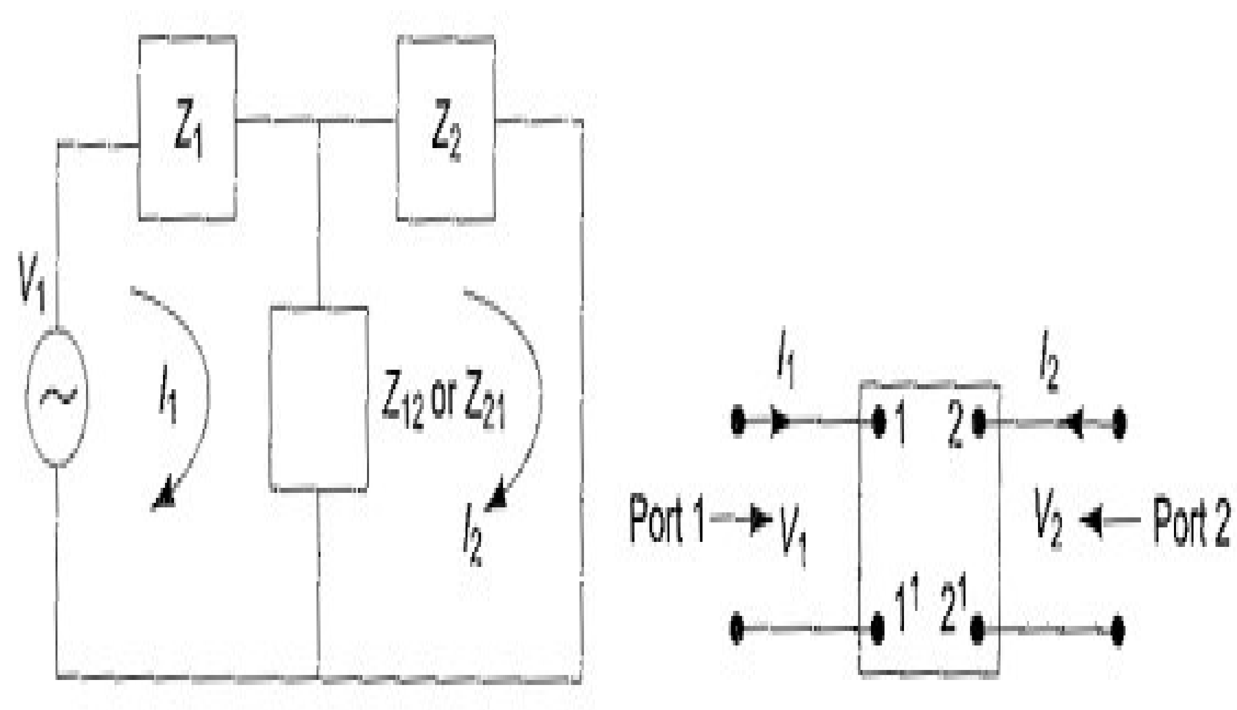

Fig. 11.2:conductively Coupled Circuit

In the circuit shown the impedance Z_{12} or Z_{21} common to loop 1 and loop 2 is called mutual impedance. It may consist of pure resistance, pure inductance, pure capacitance, or a combination of any of these elements. The general definition of mutual impedance is explained with the help of Figure 11.2 (b). The network in the box may be of any configuration of circuit elements with two ports having two pairs of terminals 1-1′ and 2-2′ available for measurement. The mutual impedance between ports 1 and 2 can be measured at 1-1′ or 2-2′. If it is measured at 2-2′. It can be defined as the voltage developed (V_2) at 2-2′ per unit current (I_1) at port 1-1′. If the box contains linear bilateral elements, then the mutual impedance measured at 2-2′ is the same as the impedance measured at 1 -1′ and is defined as the voltage developed (V_1) at 1-1′ per unit current (I_2) at port 2-2′.

Mutual Inductance of Coupled Circuits:

Mutual Inductance of Coupled Circuits:

A voltage is induced in a coil when there is a time rate of change of current through it. The inductance parameter L is defined in terms of the voltage across it and the time rate of change of current through it $\upsilon(t) = L \; di(t)/dt$, where $\upsilon(t)$ is the voltage across the coil, I(t) is the current through the coil and L is the inductance of the coil. Strictly speaking, this definition is of self-inductance and this is considered as a circuit element with a pair of terminals. Whereas a circuit element “mutual inductor” does not exist. Mutual inductance is a property associated with two or more coils or inductors which are nearby and the presence of a common magnetic flux that links the coils. A transformer is a device whose operation is based on mutual inductance (Figure 11.3).

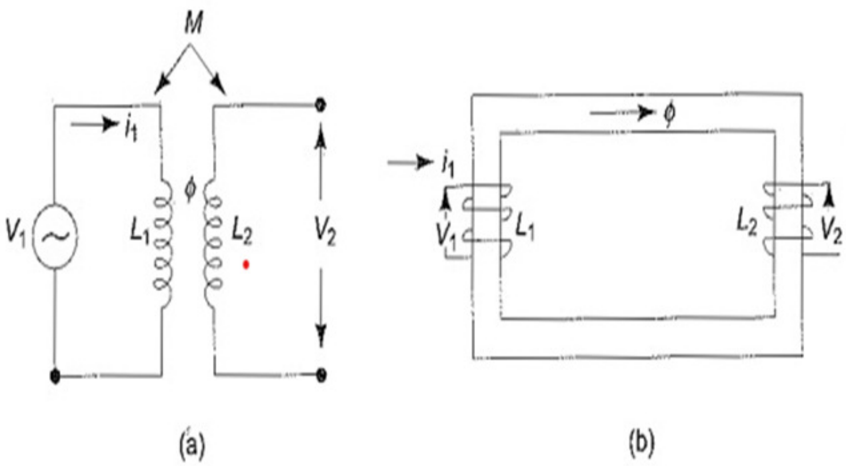

Figure 11.3 Mutual Inductance of Coupled Circuits:

Let us consider two coils, L_1 and L_2 as shown in Figure 11.3(a), which are sufficiently close together, so that the flux produced by i_1 in coil L_1 also links coil L_2. We assume that the coils do not move concerning one another, and the medium in which the flux is established has a constant permeability. The two coils may be also arranged on a common magnetic core, as shown in Figure 11.3(b).

The two coils are said to be magnetically coupled but act as separate circuits. It is possible to relate the voltage induced in one coil to the time rate of change of current in the other coil. When a voltage υ_1 is applied across L_1, a current i_1 will start flowing in this coil and produce a flux Φ. This flux also links coil L_2. If i_1 were to change concerning time, the flux 'Φ' would also change concerning time. The time-varying flux surrounding the second coil, L_2 induces an emf, or voltage, across the terminals of L_2; this voltage is proportional to the time rate of change of current flowing through the first coil L_1.

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The two coils, or circuits, are said to be inductively coupled, because of this property they are called coupled elements or Mutual Inductance of Coupled Circuits, and the induced voltage or emf is called the voltage/emf of mutual induction and is given by $\upsilon_2(t) = M_1\, di_1(t)/dt$ volts, where υ_2 is the voltage induced in coil L_2 and M_1 is the coefficient of proportionality, and is called the coefficient of mutual inductance, or simple mutual inductance.

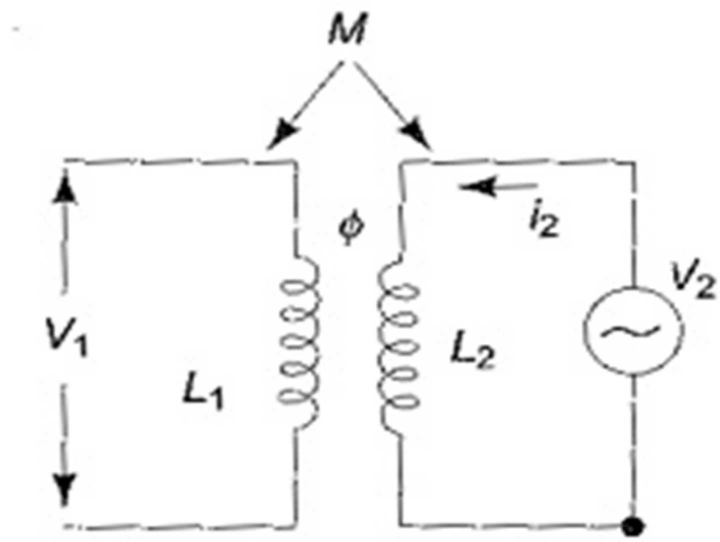

Figure 11.3(c): Current i2 is made to pass through coil L2

If current i_2 is made to pass through coil L_2 as shown in Figure 11.3(c) with coil L_1 open, a change of i_2 would cause a voltage υ_1 in coil L_1, given by $\upsilon_1(t) = M_2\, di_2(t)/dt$.

In the above equation, another coefficient of proportionality M_2 is involved. Though it appears that two mutual inductances are involved in determining the mutually induced voltages in the two coils, it can be shown from energy considerations that the two coefficients are equal and, therefore, need not be represented by two different leters. Thus $M_1 = M_2 = M$.

$$v_2(t) = M\frac{di_1(t)}{dt} \text{ Volts}$$

$$v_1(t) = M\frac{di_2(t)}{dt} \text{ Volts}$$

In general, in a pair of linear time-invariant coupled coils or inductors, a non zero current in each of the two coils produces a mutual voltage in each coil due to the flow of current in the other coil. This mutual voltage is present independently of, and in addition to, the voltage due to self-induction. Mutual inductance is also measured in Henrys and is positive, but the mutually induced voltage, M di/dt may be either positive or negative, depending on the physical construction of the coil and reference directions. To determine the polarity of the mutually induced voltage (i.e. the sign to be used for the mutual inductance), the dot convention is used.

Dot Convention in Coupled Circuits:

Dot Convention in Coupled Circuits is used to establish the choice of correct sign for the mutually induced voltages in coupled circuits. Circular dot marks and/or special symbols are placed at one end of each of the two coils which are mutually coupled to simplify the diagrammatic representation of the windings around its core.

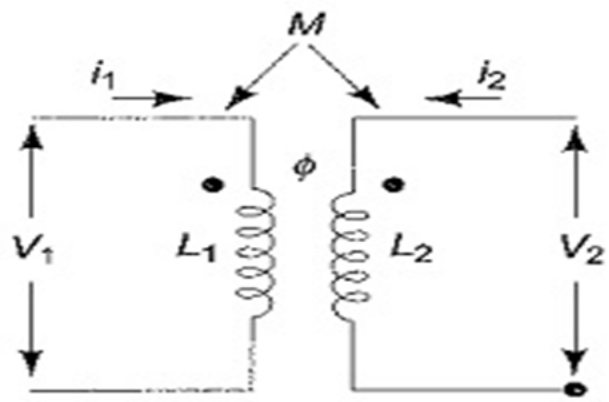

Figure 11.4 pair of linear, time-invariant, coupled inductors

Let us consider Figure 11.4, which shows a pair of linear, time-invariant, coupled inductors with self-inductances L_1 and L_2 and a mutual inductance M. If these inductions form a portion of a network, currents i_1 and i_2 are shown, each arbitrarily assumed entering at the doted terminals, and voltages υ_1 and υ_2 are developed across the inductors. The voltage across L_1 is, thus composed of two parts and is given by

$$v_1(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$$

The first term on the RHS of the above equation is the self-induced voltage due to i_1, and the second term represents the mutually induced voltage due to i_2.

Similarly,

$$v_2(t) = L_2 \frac{di_2(t)}{dt} \pm M \frac{di_1(t)}{dt}$$

Although the self-induced voltages are designated with a positive sign, mutually induced voltages can be either positive or negative depending on the direction of the winding of the coil and can be decided by the presence of the dots placed at one end of each of the two coils. The convention is as follows. If two terminals belonging to different coils in a coupled circuit are marked identically with dots then for the same direction of current relative to like terminals, the magnetic flux of self and mutual induction in each coil add together. The physical basis of the Dot Convention in Coupled Circuits can be verified by examining Figure 11.5. Two coils ab and cd are shown wound on a common iron core.

It is evident from Figure 11.5 that the direction of the winding of coil ab is clock-wise around the core as viewed at X, and that of cd is anti-clockwise as viewed at Y. Let the direction of current ii in the first coil be from a to b and increasing with time. The flux produced by i_1 in the core has a direction that may be found by the right-hand rule, and which is downwards in the left limb of the core.

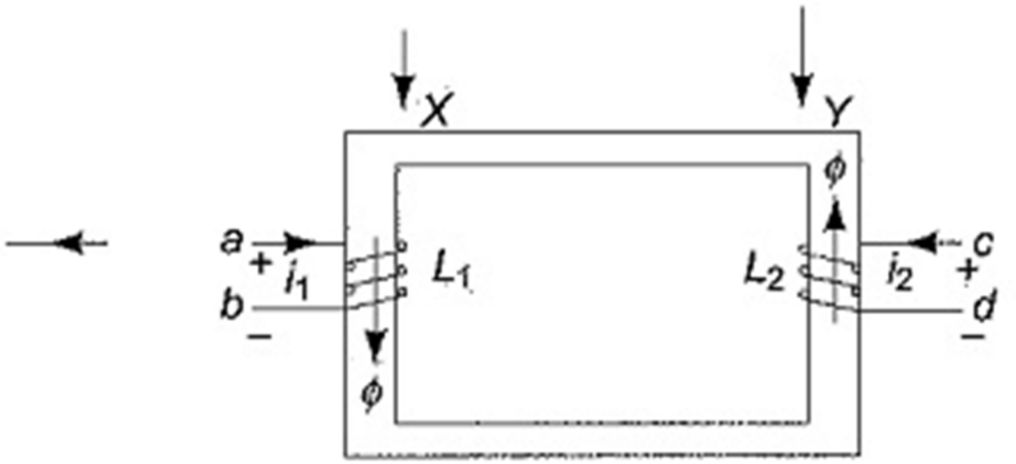

Figure 11.5: direction of the winding of coil

The flux also increases with time in the direction shown at X. Now suppose that the current i_2 in the second coil is from c to d, and increasing with time. The application of the right-hand rule indicates that the flux produced by i_2 in the core has an upward direction in the right limb of the core. The flux also increases with time in the direction shown at Y. The assumed currents i_1 and i_2 produce flux in the core that is addictive. The terminals a and c of the two coils atain similar polarities simultaneously. The two simultaneously positive terminals are identified by two dots by the side of the terminals as shown in Figure 11.6.

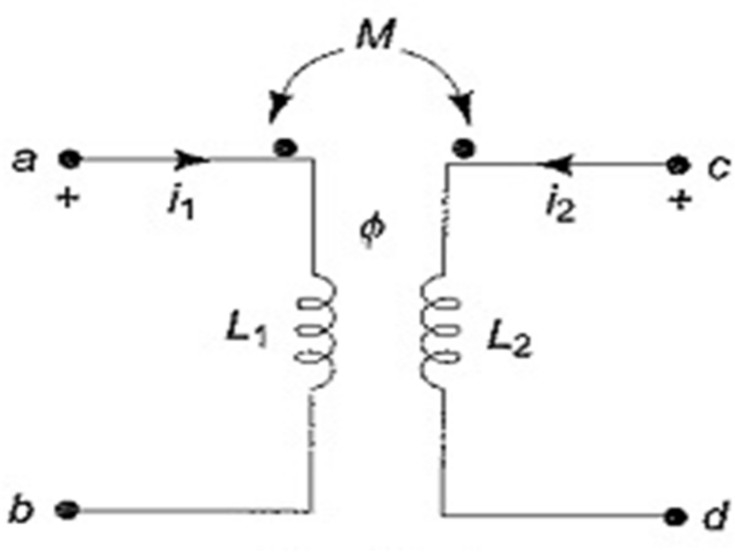

Figure 11.6: two simultaneously positive terminals are identified by two dots by the side of the terminals

The other possible location of the dots is the other ends of the coil to get additive fluxes in the core, i.e. at b and d. It can be concluded that the mutually induced voltage is positive when currents i_1 and i_2 both enter (or leave) the windings by the doted terminals. If the current in one winding enters at the dot-marked terminals and the current in the other winding leaves at the dot-marked terminal, the voltages due to self and mutual induction in any coil have opposite signs.

Coefficient of Coupling:

The amount of coupling between the inductively coupled coils is expressed in terms of the coefficient of coupling, which is defined as

where

1. M = mutual inductance between the coils
2. L_1 = self-inductance of the first coil, and
3. L_2 = self-inductance of the second coil

The coefficient of coupling is always less than unity and has a maximum value of 1 (or 100%). This case, for which K = 1, is called perfect coupling when the entire flux of one coil links the other. The greater the coefficient of coupling between the two coils, the greater the mutual inductance between them, and vice-versa. It can be expressed as the fraction of the magnetic flux produced by the current in one coil that links the other coil[7]–[10].

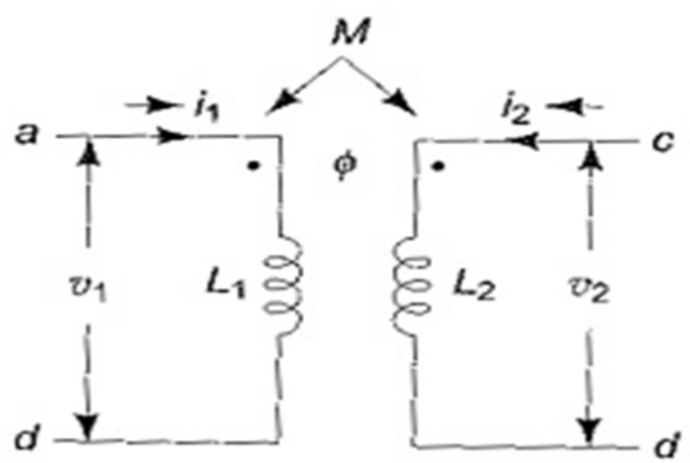

Figure 10.7: **mutually coupled circuits**

For a pair of mutually coupled circuits shown in Figure 11.7, let us assume initially that i_1 and i_2 are zero at t = 0.

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M\frac{di_2(t)}{dt}$$

$$v_2(t) = L_2 \frac{di_2(t)}{dt} + M\frac{di_1(t)}{dt}$$

Initial energy in the coupled circuit at t = 0 is also zero. The net energy input to the system shown in Figure 11.7 at time t is given by

$$W(t) = \int_0^t [v_1(t)\, i_1(t) + v_2(t)\, i_2(t)]\, dt$$

Substituting the values of $\upsilon_1(t)$ and $\upsilon_2(t)$ in the above equation yields

$$W(t) = \int_0^t \Big[L_1 i_1(t) \frac{di_1(t)}{dt} + L_2 i_2(t) \frac{di_2(t)}{dt} + M(i_1(t)) \frac{di_2(t)}{dt} + i_2(t) \frac{di_1(t)}{dt} \Big] dt$$

From this we get

$$W(t) = \frac{1}{2} L_1[i_1(t)]^2 + \frac{1}{2} L_2[i_2(t)]^2 + M[i_1(t)i_2(t)]$$

If one current enters a dot-marked terminal while the other leaves a dot-marked terminal, the above equation becomes

$$W(t) = \frac{1}{2} L_1[i_1(t)]^2 + \frac{1}{2} L_2[i_2(t)]^2 - M[i_1(t)i_2(t)]$$

According to the definition of passivity, the net electrical energy input to the system is non-negative. W(t) represents the energy stored within a passive network, it cannot be negative.

$$W(t) \geq 0 \text{ for all values of } i_1,\ i_2;\ L_1,\ L_2 \text{ or } M$$

The statement can be proved in the following way. If i_1 and i_2 are both positive or negative, W(t) is positive. The other condition where the energy equation could be negative is

$$W(t) = \frac{1}{2} L_1[i_1(t)]^2 + \frac{1}{2} L_2[i_2(t)]^2 - M[i_1(t)\, i_2(t)]$$

The above equation can be rearranged as

$$W(t) = \frac{1}{2}\left(\sqrt{L_1}\, i_1 - \frac{M}{\sqrt{L_1}} i_2 \right)^2 + \frac{1}{2}\left(L_2 - \frac{M^2}{L_1} \right) i_2^2$$

The first term in the parenthesis of the right side of the above equation is positive for all values of i_1 and i_2, and, thus, the last term cannot be negative; hence

$$L_2 - \frac{M^2}{L_1} \geq 0$$

$$\frac{L_1 L_2 - M^2}{L_1} \geq 0$$

$$L_1 L_2 - M^2 \geq 0$$

$$\sqrt{L_1 L_2} \geq M$$

$$M \leq \sqrt{L_1 L_2}$$

The maximum value of the mutual inductance is $\sqrt{L_1 L_2}$. Thus, we define the coefficient of coupling for the coupled circuit as

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

The coefficient, K, is a nonnegative number and is independent of the reference directions of the currents in the coils. If the two coils are a great distance apart in space, the mutual inductance is very small, and K is also very small. For iron-core coupled circuits, the value of K may be as high as 0.99, for air-core coupled circuits, K varies between 0.4 to 0.8

Series Connection of Coupled Inductors:

Let there be two inductors connected in series, with self inductances L_1 and L_2 and mutual inductance of M. Two kinds of Series Connection of Coupled Inductors are possible; series aiding as in Figure 11.8(a), and series opposition as in Figure 11.8(b).

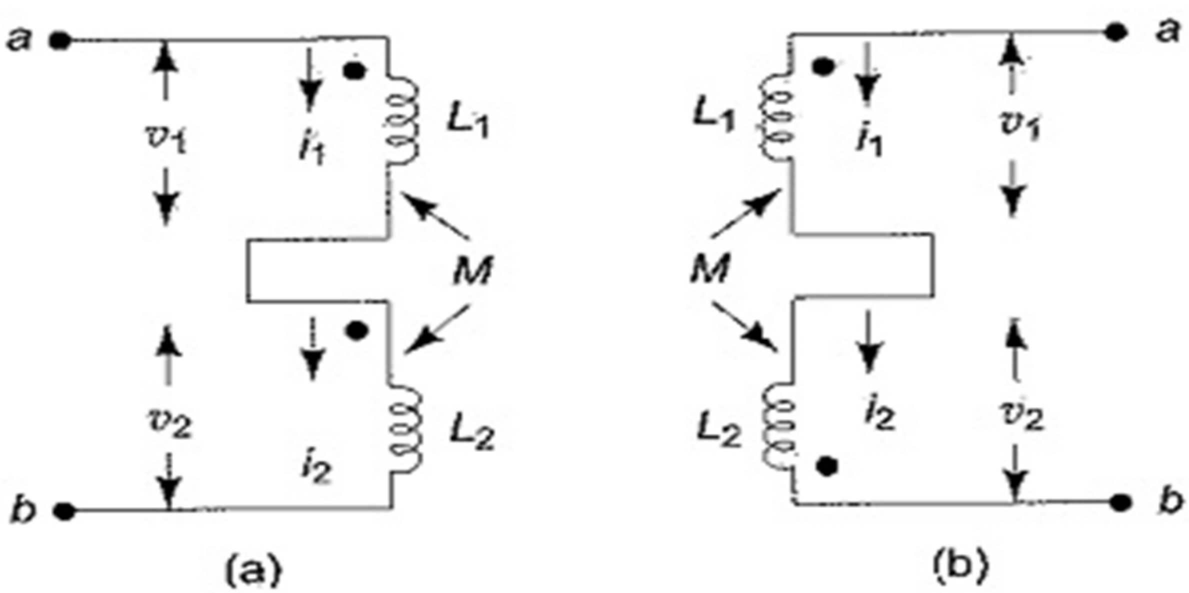

Figure 11.8: Two kinds of Series Connection of Coupled Inductors

In the case of series aiding connection, the currents in both inductors at any instant of time are in the same direction relative to like terminals as shown in Fig. 11.8(a). For this reason, the magnetic fluxes of self-induction and mutual induction linking with each element add together. In the case of a series opposition connection, the currents in the two inductors at any instant of time are in opposite directions relative to like terminals as shown in Figure 11.8(b). The inductance of an element is given by L = Φ/i, where Φ is the flux produced by the inductor.

$$\phi = Li$$

For the series aiding circuit, if Φ_1 and Φ_2 are the flux produced by coils 1 and 2, respectively, then the total flux

Two circuits are said to be 'coupled' when energy transfer takes place from one circuit to the other when one of the circuits is energized. Mutual inductance is a property associated with two or more coils or inductors which are nearby and the presence of a common magnetic flux that links the coils.

A transformer is a device whose operation is based on mutual inductance. A majority of the electrical circuits in practice are conductively or electromagnetically coupled. Certain coupled elements are frequently used in network analysis and synthesis. Transformers, transistors, electronic pots, etc. are some of these

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CHAPTER 12

NETWORK FUNCTION

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Every network is intended to harvest a precise output once input is applied to it. The network performance is refereed by learning its production for the practical input. The output is the effect of network parameters on the applied input. Thus once the network is designed, the effect of network parameters remains the same, on the input to produce the output. The effect of network parameters is mathematically expressed in the s domain which is called the network function, system function, or transfer function of the network. It is the characteristics of the network and once known for any type of applied input, the output can be predicted. Let us see the mathematical definition of the network function.

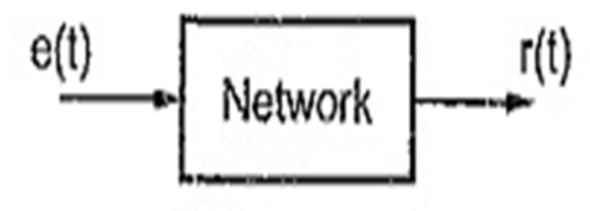

Figure 12.1: network of Function e(t) and r(t)

Consider a network shown in Figure 12.1.

e(t) = Input or excitation

r(t) = Output or response

The network function is defined as the ratio of the Laplace transform of output (response) of the network to the Laplace transform of the input (excitation) applied to the network, under the assumption that all initial conditions are zero[1]–[3].

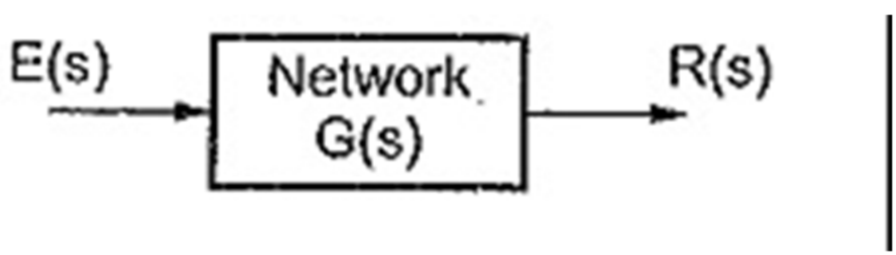

Figure 12.2: network function

The network function is denoted as H(s), G(s), or T(s) in the s domain. The network can be represented in terms of network function as shown in Figure 12.2.

Thus the network function is defined as,

$$G(s) = \frac{R(s)}{E(s)}$$

The R(s) and E(s) are the Laplace transforms of r(t) and e(t). Once network function G(s) is known, the output for any type of input can be obtained. But to determine the network function, the output terminals must be specified

Properties of Transfer Function:

The various properties of the transfer function are,

1. It is an exact model of the network explanation of the operation that the network achieves on the input to harvest the output.
2. It is the physiognomies of the network and is unique for a given set of input and output variables.
3. It is independent of the magnitude and nature of the input.
4. For any input applied, the output of the network can be predicted from the Network Function.
5. Initial conditions lose importance while obtaining the Network Function.
6. The transfer function does not provide any information regarding the physical structure of the system or network.
7. The time domain differential equation relating input and output describing the network can be easily obtained from the Network Function. Thus the Network Function gives a full description of the dynamic characteristics of the network. The excitation and the response of the network may be currents or voltages. Hence the network function G(s) may represent the impedance function, admitance function, voltage ratio transfer function, or current ratio transfer function.

1. Impedance:

When the excitation is current and the response is voltage then the network function represents the impedance function

$$G(s) = \frac{V(s)}{I(s)} = Z(s)$$

2. Admitance:

When the excitation is voltage and the response is current then the network function represents the admitance function.

$$G(s) = \frac{I(s)}{V(s)} = Y(s)$$

3. Voltage ratio transfer function:

When the excitation and response both are voltages then the network function represents the voltage ratio transfer function.

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

where V_o = Response and V_i = Excitation

Driving Point Function and Transfer Function:

For a given network, the ratio of Laplace transform of the source voltage and source current is called the driving point function. If it is a ratio of source voltage to source current, it is called the driving point impedance function denoted as Z(s) while if it is a ratio of source current to source voltage, it is called the driving point admitance function denoted as Y(s). The driving point impedance function is nothing but the input impedance of the network as viewed through the input terminals. As Y(s) is the reciprocal of Z(s), these functions are called immitance functions. While defining these functions, all initial conditions are assumed zero and there should not be any independent voltage or current sources present. The immitance function is always a ratio of two separate polynomials in 's'. Hence driving point function can be represented as,

$$\text{Driving point function} = \frac{P(s)}{Q(s)}$$

where P(s) is the numerator polynomial and Q(s) is the denominator polynomial in s.

Necessary Conditions for Driving Point Functions:

After canceling the common factors in the numerator polynomial P(s) and denominator polynomial Q(s), the necessary conditions for the driving point functions are as follows :

1. The coefficients of the numerator polynomial P(s) and the denominator polynomial Q(s) must be real and positive.
2. If poles and zeros are imaginary then such poles and zeros must be conjugate.
3. The real part of all the poles and zeros must be negative or zero and if the real part is zero, then the pole or zero must be simple.
4. There should not be any missing term between the highest and lowest degrees in the polynomials P(s) and Q(s) unless all the even or all the odd terms are missing.
5. The degree of the polynomial in numerator and denominator should differ by either zero or one.
6. The terms of lowest degree in P(s) and Q(s) may differ in degree at the most by one.

Transfer Functions:

The ratio of the Laplace transform of a network parameter at one port to the Laplace transform of a network parameter at the other port and vice versa is called the transfer function. These functions may be voltage ratio functions or current ratio functions. Such functions may also represent transfer impedance or transfer admitance of the network. These are also determined under the same assumptions as all initial conditions are zero and no dependent current or voltage source is present in the network. The transfer functions are also the ratio of two separate polynomials in 's'[4]–[6].

Necessary Conditions for Transfer Functions:

After canceling the common factors in the polynomials P(s) and Q(s), the necessary conditions for the transfer functions are as follows :

1. The coefficients of P(s) and Q(s) must be real and positive.
2. The complex and imaginary poles and zeros must be conjugate.
3. The real part of the poles must be negative or zero and if it is zero, the pole must be simple, including the origin.
4. There should not be any missing term between the highest and lowest degree of Q(s) unless all the even or odd terms are missing.
5. The polynomial P(s) may have negative terms or even some missing terms between the highest and lowest degree.
6. The degree of polynomial P(s) may be as small as zero independent of the degree of the polynomial Q(s).

Poles and Zeros of Network Function:

All the Poles and Zeros of the Network Function have the form of a ratio of two polynomials in s as,

$$G(s) = \frac{P(s)}{Q(s)}$$

The P(s) is the numerator polynomial in s having degree m while the Q(s) is the denominator polynomial in s having degree n. Hence network function can be expressed as,

$$G(s) = \frac{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \cdots + a_{m-1} s + a_m}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \cdots + b_{n-1} s + b_n}$$

Now the equation P(s) = 0 has m roots while the equation Q(s) = 0 has n roots. Thus G(s) can be expressed in the factorized form as,

$$G(s) = K \frac{(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}$$

where z_1, z_2, … z_m are the roots of the equation P(s) = 0 and P_1, P_2, … P_n are the roots of the equation Q(s) = 0.

$$K = \frac{a_0}{b_0} = \text{scale factor}$$

The z_1, z_2, … z_m, P_1, P_2 … P_n are the values of s and hence are complex frequencies as the s variable is complex.

Poles:

The values of 's' i.e. complex frequencies, which make the network function infinite when substituted in the denominator of a network function are called poles of the network function.

So values $s = p_1, p_2, \ldots\ldots p_n$ are the poles of G (s).

If such poles are real and nonrepeated, these are called simple poles. If a particular pole has the same value twice or more than that, it is called a repeated pole. A pair of poles with complex conjugate values is called a pair of complex conjugate poles.

The poles are the roots of the equation obtained by equating the denominator polynomial of a network function to zero. Such an equation is called a characteristic equation of a network.

Zeros:

The values of 's' i.e. complex frequencies, which make the network function zero when substituted in the numerator of a network function are called zeros of the network function.

So values $s = z_1, z_2, \ldots\ldots z_m$ are the zeros of G(s).

Similar to the poles, zeros also can be simple zeros, repeated zeros, or complex conjugate zeros. The zeros are the roots of the equation obtained by equating the numerator polynomial of a system function to zero.

When the order m is greater than n then there are m-n poles at infinity while if the order m is less than n then there are n-m zeros at infinity. Hence if for any rational network function, poles and zeros at infinity and zero are taken into consideration in addition to finite poles and zeros, the total number of zeros is equal to the total number of poles.

D.C. Gain of the Network:

The s variable is a complex frequency variable given by $s = j\omega$ in the frequency domain. The frequency of d.c. is zero hence the value of G(s) calculated at $\omega = 0$ i.e. $s = 0$ is called d.c. gain of the network, represented by the network function G(s). For $s = 0$, the value of G(s) is constant representing the gain of the network at zero function

Pole Zero Plot:

Pole Zero Plot: The variable s is a complex variable. Hence a complex plane is required to indicate the values of s graphically. A complex plane is a plane with X-axis as the real axis and Y-axis as the imaginary axis. The real axis is denoted as σ axis while the imaginary axis is denoted as $j\omega$ axis. Such a complex plane used to indicate values of s in it is called an **s-plane**. All the poles and zeros can be easily indicated in such an s-plane[7]–[10].

The poles are indicated by the symbol 'cross' (X), in the s-plane.

The zeros are indicated by the symbol of 'small circle' (O) in the s-plane.

The plot obtained by locating all the poles and zeros of the network function in the s-plane is called the Pole zero plot of that network function.

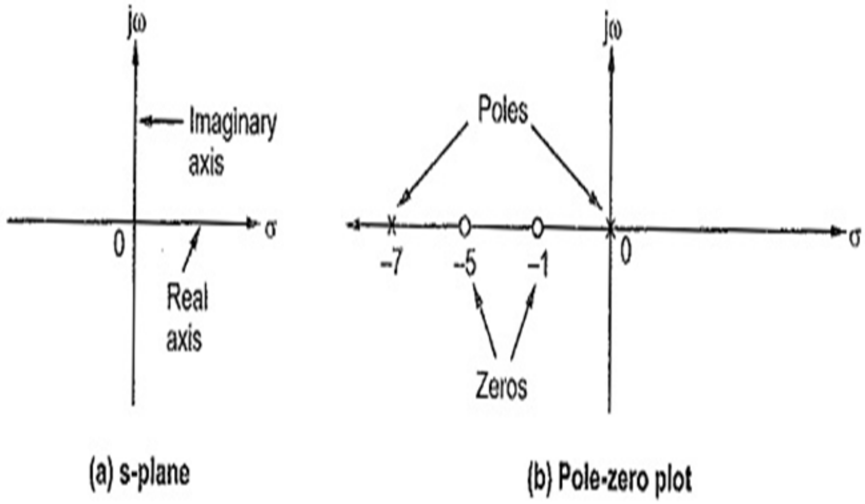

Figure 12.3: s-plane whose pole-zero

The s-plane is shown in Figure 12.3(a).

While considering

$$G(s) = \frac{20(s+1)(s+5)}{s(s+7)}$$

whose pole-zero plot is shown in Figure 12.3(b).

The scale factor of a network function can not be indicated on the pole-zero plot. Thus while obtaining a network function from the pole-zero plot, additional information is necessary to obtain the scale factor of a network function. For the repeated poles or zeros, the marks equal to the number of repeated poles or zeros must be shown in the pole-zero plot.

Significance of Poles and Zeros:

The network function is always the ratio of the Laplace of the output to the input. Thus if the input is known, it is easy to obtain the expression for the Laplace transform of the output variable, from the network function.

Thus if,

$$G(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

Then,

$$V_{out}(s) = G(s) \times V_{in}(s)$$

Now to find the response in the time domain, it is necessary to find the inverse Laplace transform of $V_{out}(s)$, which is possible by expanding it in partial fraction expansion form.

It is important to note that the partial fraction expansion depends on the denominator of the Laplace transform of the output variable i.e. the poles of the output function. If F(s) is the output variable then for each real pole of F(s), there exists an exponential term in the time domain.

$$F(s) = \frac{K}{(s+a)} \Rightarrow f(t) = K\, e^{-at}$$

For a quadratic pole with purely imaginary roots, as the poles of F(s), there exists a sine or cosine term in the time domain.

$$F(s) = \frac{N(s)}{(s^2+\omega^2)} \Rightarrow f(t) = K_1 \sin \omega t \text{ or } K_2 \cos \omega t \text{ or both}$$

similarly a quadratic with complex conjugate roots as the poles of F(s) produce damped sine or damped cosine type of terms in the time domain. While the partial fraction coefficients are decided by the zeros of the function F(s). Thus the poles determine the waveform of the time variation of the response while the zeros determine the magnitude of each part of the response. We have seen that driving point functions can be the impedance Z(s) functions or admitance Y(s) functions. Hence these are called immitance functions.

If the driving point function is an impedance function i.e.

$$Z(s) = \frac{V(s)}{I(s)}$$

Then zero of Z(s) means zero voltage for a finite current i.e. short circuit condition in practice.

While a pole of Z(s) means zero current for a finite voltage i.e. open circuit condition in practice.

Hence for poles, a one-terminal pair network acts as an open circuit while for zeros it acts as a short circuit.

As the poles and zeros are critical frequencies, practically poles and zeros indicate the frequencies for which the network acts as an open circuit or short circuit.

To understand this, consider the transform impedance of a single capacitor as Z(s) = 1/sC. It has a pole at s = 0 thus for a zero frequency i.e. at s = 0, Z(s) = ∞ i.e. it acts as an open circuit. While this Z(s) has a zero value for s = ∞ i.e. a zero at s = ∞ for which it acts as a short circuit.

Frequency Domain Analysis:

Frequency Domain Analysis – As we know already, the responses of the networks to the various time-dependent inputs such as step, ramp, exponential, etc. are studied. Let us now discuss the response by the network to a purely sinusoidal frequency-dependent input.

Consider a system with network function H(s). The input applied is purely sinusoidal and frequency dependent.

$$\text{Input} = A \sin \omega t$$

where

ω = input frequency

The ω is variable and can be changed from 0 to ∞.

The response given by the network, when input is purely sinusoidal and its frequency ω is changed over a certain range is called the frequency response of the network.

In other words, the steady-state response of a network to a variable frequency purely sinusoidal input is called the frequency response of a network.

Frequency Domain Network Function:

To study the effect of change in ω on the network function, it is necessary to obtain the frequency domain network function from the s-domain network function.

The network functions which describe the behavior of the networks in the sinusoidal steady state can be obtained by replacing complex frequency variable 's' with 'jω'. Thus if H(s) is the s domain network function, then the frequency domain network function is denoted as $H(j\omega) = H(s)|_{s=j\omega}$.

$$\therefore \quad H(j\omega) = H(s)|_{s=j\omega} = \text{Frequency domain network function}$$

As variable s is a complex variable, the frequency domain network function is also complex.

Frequency Domain Network Function:

As the frequency domain network function is complex, it can be expressed mathematically in two ways i.e. using rectangular coordinates or using polar coordinates.

In rectangular coordinates, it can be writen as,

$$H(j\omega) = R(\omega) + j\,X(\omega) \qquad \ldots (1)$$

R(ω) = Re[H(jω)] = Real part of network function

X(ω) = Imj [H(jω)] = Imaginary part of network function

While in polar coordinates H(jω) can be writen as,

$$H(j\omega) = |H(j\omega)| \angle \phi(\omega) \qquad \dots (2)$$

$|H(j\omega)|$ = Magnitude of the network function

$\Phi(\omega)$ = Angle of the network function

The equations (1) and (2) are related to each other as,

$$|H(j\omega)| = \sqrt{[R(\omega)]^2 + [X(\omega)]^2}$$

$$\phi(\omega) = \tan^{-1} \frac{X(\omega)}{R(\omega)}$$

Sometimes the magnitude $|H(j\omega)|$ is denoted as M_R while the angle of $H(j\omega)$ is denoted as Φ_R so polar representation is expressed as,

$$H(j\omega) = M_R \angle \phi_R$$

where

M_R = Resultant magnitude which is the function of ω

Φ_R = Resultant phase angle which is the function of ω

Thus $R(\omega)$, and $X(\omega)$ are the parts of the network function in the rectangular coordinates while M_R and Φ_R are the parts of the network function in the polar coordinates.

The frequency response means to sketch the variations in the various parts of the network function as the ω is changed from 0 to ∞. The use of polar coordinates is very common to obtain the frequency response. These parts of network function play an important role in network design because,

1. Most of the time, the specifications for which the networks are to be designed, are given in terms of magnitude and phase. The real and imaginary parts are rarely used to describe the required specifications.
2. The measurement of these parts of the network functions is easy due to availability of the instruments like C.R.O., voltmeters, ammeters, watmeters, etc.

The concept of complex frequency, physical interpretation of complex frequency, transform impedance and transform circuit, series and parallel combination of the element, terminal pairs or ports, network function for the one port and two port, poles and zeros of network function, significance of poles and zero's, Every network is designed to produce a particular output when input is applied to it. The network performance is judged by studying its output for the applied input. The output is the effect of network parameters on the applied input. Thus once the network is designed, the effect of network parameters remains the same, on the input to produce the output. The effect of network parameters is mathematically expressed in the s domain which is called the network function, system function, or transfer function of the network. It is the characteristics of the network and once known for any type of applied input, the output can be predicted. Let us see the mathematical definition of the network function.

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Questionnaire:

1. What will be the effect of using capacitor for power factor correction in a single phase circuit? ...
2. Explain current, voltage and power?
3. Comment upon the pole and zero of voltage transfer function of a simple RC integrator:
4. A network which contains one or more than one source of e.m.f. is known as

5. In non-linear network does not satisfy
6. A closed path made by several branches of the network is known as:
7. The number of independent equations to solve a network is equal to:
8. A network consists of linear resistors and ideal voltage source. If the value of the resistors are doubled then voltage across each resistor is;
9. A network has 4 nodes and 3 independent loops. What is the number of branches in the network ?
10. Pole of a network is a frequency at which:

References Book for Further Reading

1. Fundamentals of Electric Circuits by Charles K.
2. Network Theory: Analysis and Synthesis by Ghosh.
3. Network Analysis by Valkenburg. ...
4. Engineering Circuit Analysis by William H.
5. Circuit Theory - Analysis and Synthesis by Abhijit Chakrabarti.