CONTROL SYSTEM

Tirumala Vasu G
Dr. Abhishek Kumar Sharma

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CONTENTS

CHAPTER 1

FUNDAMENTALS OF CONTROL SYSTEM

Mr. Tirumala Vasu G, Assistant Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- tirumala.vasu@presidencyuniversity.in

A system is a combination of diverse components that function together as a collective entity to achieve a given activity. The physical system is described as the aggregate activity of physical items in a system to fulfill a purpose, for example a classroom. The classroom with seats, tables, fans, lighting, chalkboard, etc., together constitutes a physical system. A kite is made out of sticks and paper, as well as the water in a lake is likewise a form of physical system.

Control system

Control implies to govern or direct. Thus, a control system is the interconnection of numerous physical parts coupled in a manner to govern or steer itself or the other system. Let's examine a finest example utilized in our day-to-day lives, Air conditioner. It accepts the user's input via remote, executes the instructions, and provides the air to the relevant room. The temperature control mechanisms installed in the air conditioner regulates the temperature according to the user's needs. When the specified temperature achieved, air conditioner automatically cuts off the compressor. As soon as the temperature begins fluctuating, it again turns on the compressor. The settings may be done manually through remote. Air conditioner features three controls, temperature control, humidistat, monitoring air stats. The thermostat regulates the temperature, the humidistat controls overall relative humidity, and air stats regulates the airflow within the room. So, we may infer that a control system is indeed an interconnection of the physical components to deliver the intended function with some regulating action. Now, let's review some basic words that will be beneficial[1]–[5].

Plant

The component of the system that is to be managed or regulated is known as plant of the process. In a control system, it is commonly referred as a transfer function, which establishes the connection between both the input and output of the system without feedback. It indicates that plant may be anything that receives an input and gives the output. The plant may have one or multiple output and inputs. The sensors are used to monitor the plant's output, whereas actuators control the plants inputs. The architecture of the plant is presented inFigure 1.1:

Figure 1.1: Illustrates the architecture of the plant.

The input variable in a system is often termed reference input and the output is known as the controller output.

Controller

The controller is the component of the system. It might also lie exterior to the system. The job of the controller would be to control the plant or process. Every system receives an input but also defines the output after studying the nature of the input. The controller inside the control system is indeed a mechanism that lowers the difference between the current value and intended value of the system. Therefore, the actual value denotes the genuine value, whereas desired value is indeed the set-point or goal value.

Input

It is a signal from of the external energy source delivered to the control system to generate the intended output and it represents the desired action that is capable of causing any reaction in a system. The primary forms of input employed inside the control system are SISO (Single Input Single Output) and MIMO (Multiple Input and Multiple Output) (Multiple Input and Multiple Output). SISO denotes that the system provides one output for the single input, whereas MIMO creates multiple outputs again for multiple inputs. It is displayed below in Figure 1.2:

Figure 1.2: Illustrates the signal of input u1 and output y1.

The reference input inside a control system also was known as the set-point, the intended value. It functions as the foundation for error-controlled regulation employing negative feedback towards error control.

Output

It is a genuine response to the applied input signal from the control system. The inputs are stimulated into the system as well as the outputs are the processed outcomes of those inputs. The outputs are the outcomes of either a tiny element of the process or the full operation.

Disturbances

Disturbances are a form of signal which negatively affects the output value of the control system. The disruptions might be internal or external. The internal disturbances that originate in the system itself while the external disturbances are created outside the system. Such disruptions operate as an additional input to the network and the usual input and further alter the system's output.

Terminology of a control system

1. The terminology of the control system were classed as:
2. Automatic control system
3. Manual control system
4. Linear control system
5. Time-variant control method

6. Time-invariant control method

Automatic control system

An automated control system without even any human interaction is known as an automatic control system, such as an auto-pilot control system. It is a sort of dynamic system wherein differential equations commonly explain processes. Other examples of the automated control system include freezers, automatic ticket machines, etc. Because of the use of feedback, the closed-loop system enables the system to rectify the disturbances in the output, which makes the process an automated control system.

Manual control system

A control system that is governed with human interaction is known as a manual control system. It specifies the manual controls that a person outside the system does. Examples include sign-off paperwork and bank reconciliation. The fundamental role of manual control would be to disturb or change the process. The Control system may either be automated or manual, or both. Automatic controls were essential when the system includes large transactions with similar kind. The manual controls were based on the needed judgment.

Linear control system

As the name says, it describes the linear connection between the input and output. Or A system that has the input and output connection is defined by the linear difference equation is referred to as a system. Such systems also obey the superposition principle.

Time-variant control system

A system in which both the input and output connection is characterized by the difference equation with changing coefficient is described as a time-variant system.

Time-invariant control system

A system where the input and output connection is characterized by the difference equation with constant coefficient was known as a time-invariant system.

Negative feedback

The block diagram of such a closed-loop system is depicted below (Figure 1.3):

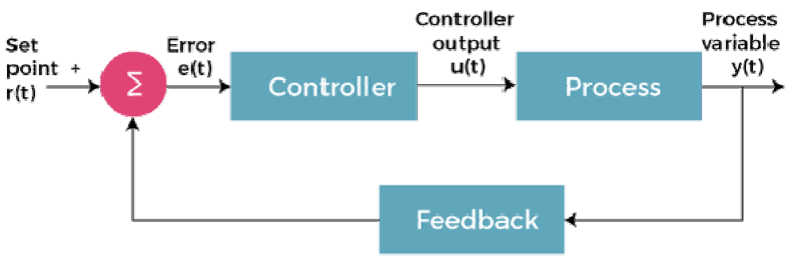


Figure 1.3: Illustrates circuit diagram of closed-loop system.

When the input is stimulated to the controller, it creates an actuation signal which regulates the plant. The output, in such a circumstance, adjusts automatically until the required response is attained. The feedback allows the system to rectify the changes in the output and eliminate mistakes from it. Hence, a closed-loop system is sometimes termed an automated closed-loop system.

Almost all are acquainted with the feedback route in the control system. The feedback channel assists in assessing the mistake. The transfer function best reflects the connection between the input and output and so assists in assessing the mistake via the feedback.

Basic ideas of a control system

A control system defines the connectivity between numerous components. The separate components of a system might be electrical, hydraulic, mechanical, thermal, or chemical in nature. A properly designed control system tends to offer the optimal response for the overall system. It can also manage the external, internal, and time-dependent disruptions successfully. The essential ideas of a control system are: To minimize the mistake, and to decrease the time-response. The fewer the difference between the actual value and the intended value, the better the system's reaction will be. It happens because no system wants any fault in between. The minimal time responsiveness of the system helps to load modifications to the system[6], [7].

Example let's explore an example of water-level control system inside a tank. When the pump is turned ON it permits the flow of water into to the tank. Once the tank has been filled up towards the specified level, the pump will turn OFF. The schematic diagram of the water-level control system can be seen below Figure 1.4:

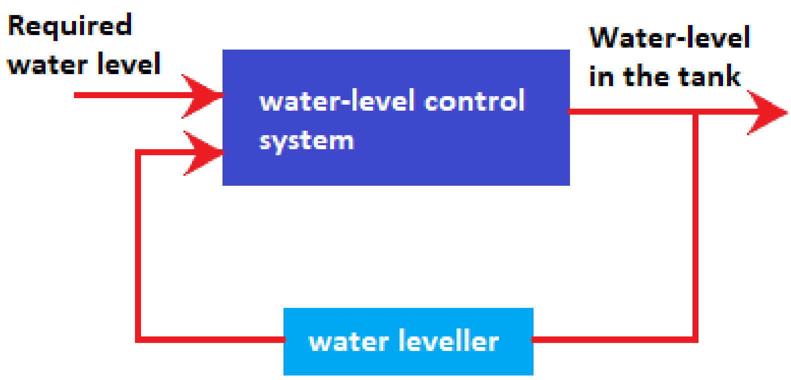


Figure 1.4 water level control system

Some individuals utilize a random approach that indicates the full tank and manually changes the ON and OFF button. But, in the sectors and workplaces where there are numerous tanks and enormous systems, the procedure operates automatically. The sensors deliver signals to the system. The water-level sensor delivers the signal towards the system that informs the liquid level existing in the water tank. The system compares this level with the desired water level. The system further gives the right reaction to get the needed tank level of water. If the level of water is less than the specified value, it turns ON the pumps and water from the inflow flow into the tank. It is an example of the feedback control system in which the sensor signals are provided feedback from the output. It examines the actual output of the system with the necessary value and adjusts appropriately.

Through the use of feedback inside a control system, the system displays decreased susceptibility to the undesirable internal and external disturbances.

Feedback loop

We understand that in a control system this same controller creates the needed signal according to input. However, we have already stated earlier, that control systems are generally classed as: Open-loop control system and Closed-loop control system.

In an open-loop control system, the current input is independent of the previously created output. Thus it is evident that here feedback loop is not existent, since no signal is given back to the input for further processing. So, this causes the formation of such a system within which there exists a high probability of attaining such an output that displays divergence from the intended output. Thus in order to create a system that produces a desired outcome, feedback mechanisms are employed. Basically, a system combined with a feedback loop is called as a closed-loop control system. In this type of system, a part of the output is fed back to the input. Further, the created output is compared with the existing input and based on the fluctuation, the controller creates the signal for obtaining the required value. In the absence of a feedback mechanism, the output and input display non-interdependency. This indicates that in such system, the current output does not impose any impact on future output values[8], [9].

As we have previously established that the usage of feedback in a system, minimizes the likelihood of fluctuation inside the system parameters due to undesirable disturbances. This is thus because, under varying circumstances, the values of the system's parameters may display fluctuations.

And such variances might lead to produce severe consequences on the functioning of the system. Thus the feedback is incorporated into the system, in order to render it insensitive to undesirable changes in the parameter. Majorly electronics circuitry like amplifiers, synthesizers, and etc. find applications of the feedback loops.

Schematic Representation of Feedback System

The graphic shown displays the block diagram of the control system with feedback in Figure 1.5:

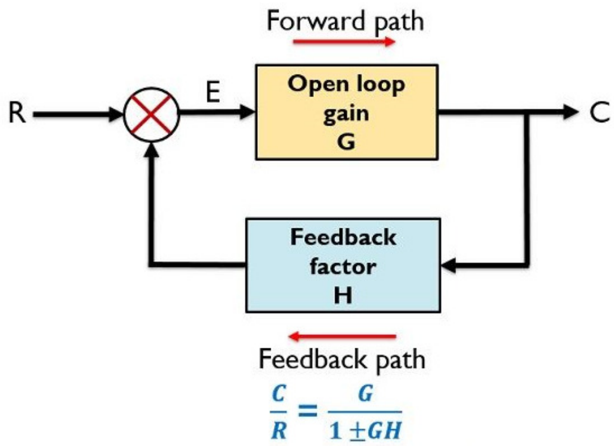


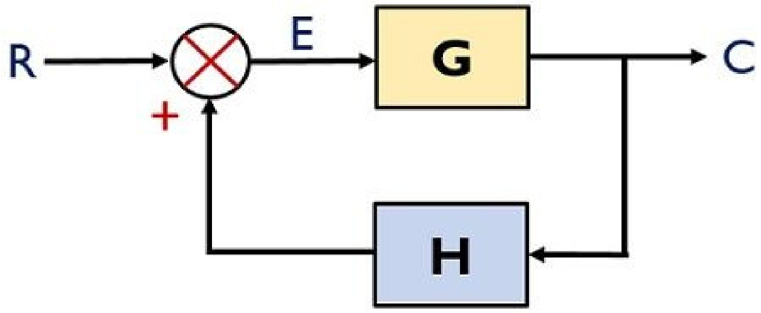
Figure 1.5: Illustrates the control system with feedback.

The key important components of a feedback system are detecting, regulating and actuating the process within the system. More precisely, the reasons for integrating feedback in any electrical circuit are as follows: Feedback affects the gain and also the reaction of the system. The usage of feedback brings the independence of the system’s features with the change in operating circumstances like applied voltage and fluctuating temperature. The non-linearity of a components included in the system tends to generate a considerable decrease in signal distortion.

Types of Feedback Systems

In every control system, feedback may be supplied in primarily two ways. Thus feedback is often categorized as:

Positive Feedback: A positive feedback system entails a situation in which the feedback signal is already in phase with both the reference input signal. Thus the two get merged and combine the resulting answer serves as input for subsequent system functioning in Figure 1.6.



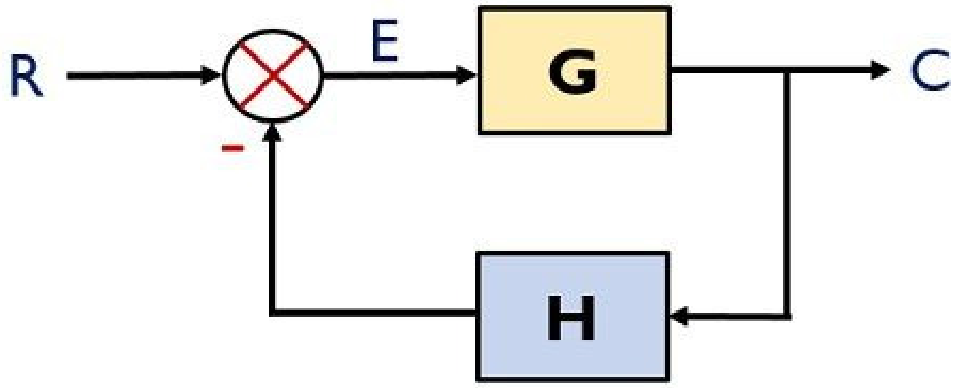
$$\frac{C}{R} = \frac{G}{1 - GH}$$

Figure 1.6: Illustrates the positive feedback loop with input output signal.

As the two signals are joined to generate the resulting response the case of a positive feedback system, therefore this raises the total gain of the system.

As the size of the input signal gets rises in the case of positive feedback, hence leading to create oscillatory response inside the system.

Negative Feedback: The system that occurs when the feedback signal is not in phase with both the reference input generally referred to as a negative feedback system. Due to out of phase connection, the two signals were subtracted throughout order to obtain the difference signal in Figure 1.7.



$$\frac{C}{R} = \frac{G}{1+ GH}$$

Figure 1.7:Illustrates the negative feedback loop with input output signal

The inclusion of negative feedback gives rise to a decrease in the total benefit. As the difference in the two values represents the error value which is required to be compensated in order to get the intended value. Thus it is usually advised that the system will have a low value of error signal.

As such systems give higher stability and circuit responsiveness with an increase in operational bandwidth. Thus most of the control systems employ negative feedback so as to have a decrease in the total gain. For a better understanding of the functioning of two kinds of feedback systems, we might take an example of a room heater. When positive feedback is applied in a room heating system, subsequently when the output temperature is high and it is delivered to the reference input then the two will be combined and this will lead to create a further rise in the input signal. Hence the temperature will be elevated higher over the standard value. And if the temperature gets below the prescribed value then it will lead to trigger the shutting off the system. Therefore, a broad category one control system involves negative feedback. Now let us learn, how the parameters display fluctuation in a control system by adding the feedback.

Effects of Parametric Variations on Output

Let us first explore the open-loop control scheme:

As we know that the transfer function of a system is specified as output by input. Thus for this system, it is given as:

$$G(s) = \frac{C(s)}{R(s)}$$

Thus

$$C(s) = G(s).\,R(s)$$

Suppose due to variation in parameter, ΔG(s) is the change that gets introduced in the gain, thus the variation in the output will be given as:

$$C(s) + \Delta C(s) = [G(s) + \Delta G(s)]\ R(s)$$

$$C(s) + \Delta C(s) = G(s)\ R(s) + \Delta G(s)\ R(s)$$

Since we know

$$C(s) = G(s).\,R(s)$$

Thus on substituting this in the above equation, we will get

$$C(s) + \Delta C(s) = C(s) + \Delta G(s)\ R(s)$$

Hence

$$\Delta C(s) = \Delta G(s)\ R(s) \quad \text{——— eq 1}$$

This is the change in output of the system, due to change in the transfer function of the system because of parameter variation in an open-loop control system.

Suppose we have a closed-loop control system in Figure 1.8:

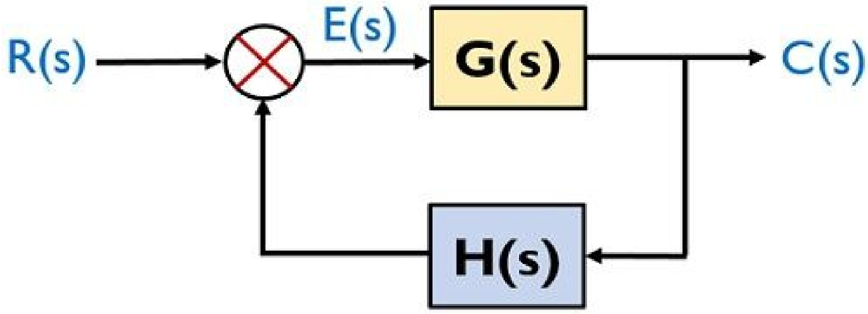


Figure 1.8: Illustrates the closed-loop control system.

For a closed-loop system with negative feedback, the transfer function is given as:

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Let ΔG(s) be the change in the transfer function, due to parameter variation, thus the change in output will be given as:

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + [G(s) + \Delta G(s)]\, H(s)} \,.\, R(s)$$

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s)H(s) + \Delta G(s)H(s)} \,.\, R(s)$$

Since ΔG(s)H(s) is very small in comparison to G(s)H(s). Thus it can be neglected from the denominator.

$$C(s) + \Delta C(s) = \frac{G(s) + \Delta G(s)}{1 + G(s)H(s)} \,.\, R(s)$$

$$C(s) + \Delta C(s) = \frac{G(s)}{1+G(s)H(s)} \,.\, R(s) + \frac{\Delta G(s)}{1+G(s)H(s)} \,.\, R(s)$$

On substituting

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} \,.\, R(s)$$

Therefore,

$$C(s) + \Delta C(s) = C(s) + \frac{\Delta G(s)}{1+G(s)H(s)} \,.\, R(s)$$

Further

$$\Delta C(s) = \frac{\Delta G(s)}{1 + G(s)H(s)} . R(s) \quad \text{eq 2}$$

This describes the change in output due of the variation in the parameter.

Since the magnitude value of 1 + G(s)H(s) is significantly bigger than unity. Thus on comparing eq1 with eq2, it is seen that owing to parameter fluctuation in case of closed-loop system overall change is output is decreased by factor 1 + G(s)H(s) (s). This is the result of the existence of feedback inside the system.When using the open-loop system, since the feedback is missing hence such a decrease is also absent.

Open-loop Control System

An open-loop system is a sort of control system in which the output of the system relies on the input however the input or even the controller is independent of an outcome of the system. These systems do not feature any feedback loop and so are also referred to as non-feedback system. In open-loop systems, because output is neither monitored nor given back to the input for the further assessment.

Open-loop System

We know that a control system guides the functioning of a system in order to carry out a given aim.

Everything surrounding us that gives an output demands effective regulating. Like from a compressor, TV, refrigerator to antennas etc. everything requires regulating, are therefore control systems.

In the open-loop control system, a reference input is supplied to the system in order to produce the desired output. But the produced output is not evaluated by the computer for subsequent reference input.

The graphic shown illustrates the block diagram of an open-loop control system inFigure 1.9:

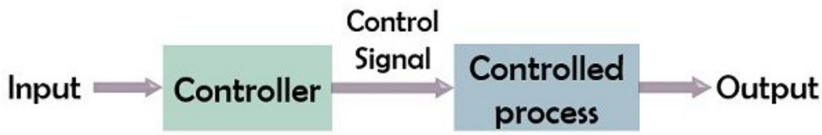


Figure 1.9: Illustrates the block diagram of an open-loop control system.

Here as we can see that the system consists of two blocks, one is the controller while other is controlled process.

Basically, according to the required output, an input is provided to the controller of the system. Depending on the achieved input, the controller generates the control signal which is fed to the processing unit. Thus according to the control signal, proper processing is performed and output is achieved.

But as there is no feedback path present in the system, thus whether the achieved output is desired or not the input has nothing to do with it.

So, this is the reason we say that in an open-loop system the input is independent of the output.

It is noteworthy here that this generally produces an error in the system because there exist no chances to adjust the input when the output shows variation from the expected value.

Example

We all are aware of the operation of a traffic light controller present at various road crossings.

Basically the three signals generated by the controller is time-dependent. An internal timing is provided to the controller at the time of designing the system. So, when the traffic signal controller is installed at the crossing then each signal is displayed by the controller timely, independent of the rush present on any of the side.

Here the system has nothing to do with the generated output as it is not changing its input according to the traffic present on any particular side or any other factor. Simply after a definite time interval, according to the initially provided input, the system is generating the output.

Basically relays are used to provide timing sequence to the system.

So, this clearly indicates that whatever output is achieved, the input will remain independent of it.

Consider below:

We know transfer function is given as:

$$G(s) = \frac{output}{input}$$

When we separately consider the transfer function of each block then it will be given as:

$$G_1\ (s) = \frac{x_1}{x_i}$$

$$G_2\ (s) = \frac{x_2}{x_1}$$

$$G_3\ (s) = \frac{x_o}{x_2}$$

Thus the overall transfer function will be:

$$G_1 * G_2 * G_3 = \frac{\cancel{x_1}}{x_i} * \frac{\cancel{x_2}}{\cancel{x_1}} * \frac{x_o}{\cancel{x_2}}$$

So, the open-loop gain will be:

$$G = \frac{x_o}{x_i}$$

The usage of the open-loop control system shows that the operator of the system is willing to contemplate some little divergence in the output from the intended value.

Advantages of Open-loop Control System

These systems feature simplicity in design and simple maintenance, Due to the reduced number of units, fundamentally the system is inexpensive, the output produced by the system demonstrates stability, and the operation is relatively comfortable.

Disadvantages of Open-loop Control System

These systems need periodic recalibration, the systems are more prone to faults, the alterations in the intended output might be the consequence of internal or external disruptions.

Applications

Open-loop systems commonly find their applications in the following areas are found in the traffic light controlling system, TV remote control, Immersion Rod, Automatic washers and dryers, in room heaters, Automatic door opening and shutting systems etc. So, us may deduce that the open-loop system never employs a feedback loop in the circuit because it has nothing to have with the output for additional reference input.

Closed-Loop Control System

A closed-loop control system is a sort of control system in which the regulating action demonstrates dependent on the produced output of the system. In simple terms, in these systems, the output produced by the system regulates the input provided to the system.

The modification in input in response to the outcome helps to provide more consistent system output. Thus controllability inside the closed-loop system is obtained by the output created by adopting a feedback route.

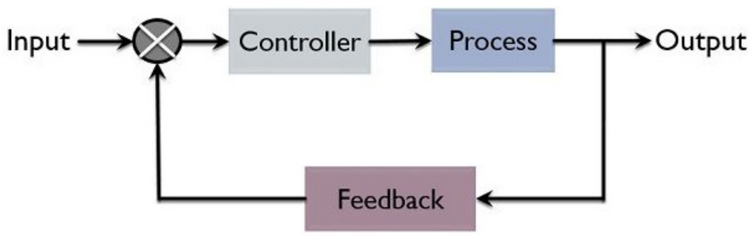


Figure 1.10: Illustrates the closed-loop system is obtained by the output created by adopting a feedback route.

Closed-loop systems are classified as completely automated control system since it is constructed in a manner that the attained output is automatically compared with the reference input to achieve the needed output.

Need for Closed-Loop Control System

We have previously covered in our previous post about the control system. A control system is an apparatus that is meant to create a given output by the action of needed controls.Now the controls offered to the system may be either output independent or output dependent. This variance leads to produce two separate sorts of control system.

A system wherein the controlling action was independent of the created system output was known as open-loop control system. When using a closed-loop system, the generated output governs the functioning of the system via the employed of feedback.Basically a closed-loop system was created to address the drawbacks associated with just an open-loop system.

Researchers know that open-loop systems do not contain the capacity to automatically produce correct output.We all are aware of the fact that the fundamental need of operating an electric or electronic system is to provide the required output. And in any system, if the measurements is not completed and the needed output is not produced then it becomes practically hard to acquire the precise system response.So, to get the correct system reaction the simplest technique is to compare the applied input with accomplished output. This assists in detecting the mistake which is there within the system. Therefore, after the error is measured then it may be decreased to the lowest feasible amount in order to produce the desired result.Thus in a closed-loop system, a feedback signal is sent to the input. This feedback signal as well as the reference input signal functions as system stimulation to produce the desired response. Thus in this manner, the output performs the controlling activity in a closed-loop system.

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CHAPTER 2

FEEDBACK

Dr. Sumantra Chaudhuri, Assistant Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- sumantra.c@presidencyuniversity.in

Till now we have employed the phrase feedback so very many times. But one must clearly realize what feedback actually implies. So simply feedback functions as the feature of the system that permits comparison between accomplished output and reference inputs of a system.A feedback is often a component of the output signal which would be provided back to the input signal to ensure that the two can be evaluated and the intended output may be reached if the current output displays variance with the desired output. Thus feedback loop is regarded as the main parameter of the closed-loop control system (Figure 2.1).

Feedback in just about any circuit may be typically of two sorts:

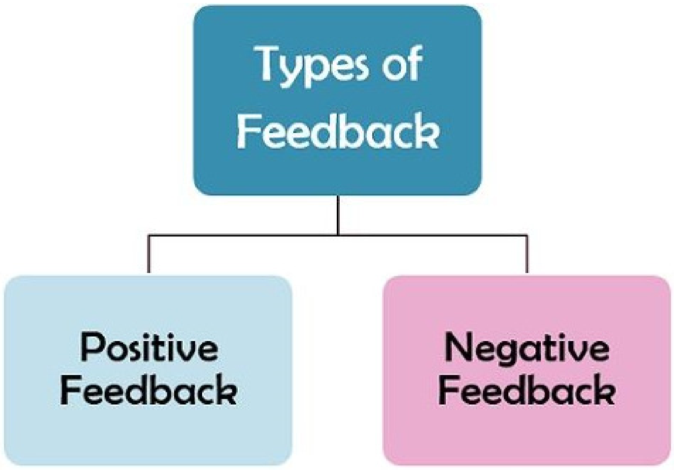


Figure 2.1: Illustrates the types of feedback in control system.

Positive Feedback: The form of feedback in a control system in which the input signal as well as the feedback signal are already in phase with each other is known as either a positive feedback system.Within those systems, the reference input is coupled with the feedback signal therefore boosting the gain of the whole system.Negative Feedback: Inside the case of negative feedback, both input signal as well as the feedback signal display out-of-phase interaction w.r.t each other.Thus the applied input signal as well as the feedback signal were subtracted to yield the error signal. This results to a decline in the total gain of the system. Thus one may say that it is the element that is most crucially accountable to get the desired reaction of a system[1]–[6].

Operation of the a Closed Loop System

The graphic shown depicts the comprehensive block diagram depiction of a closed-loop control system in Figure 2.2:

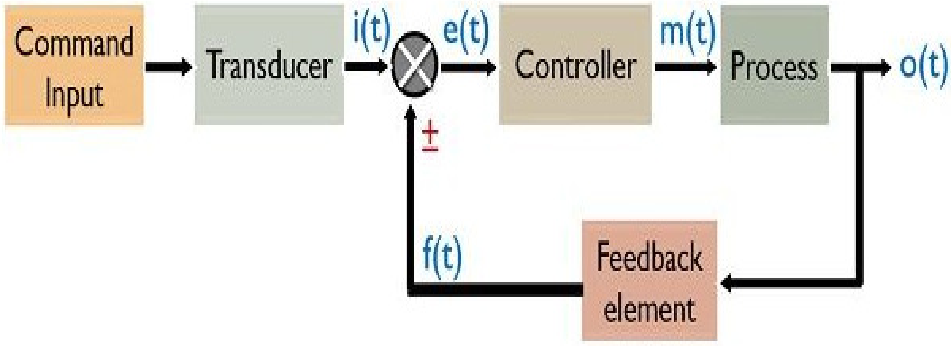


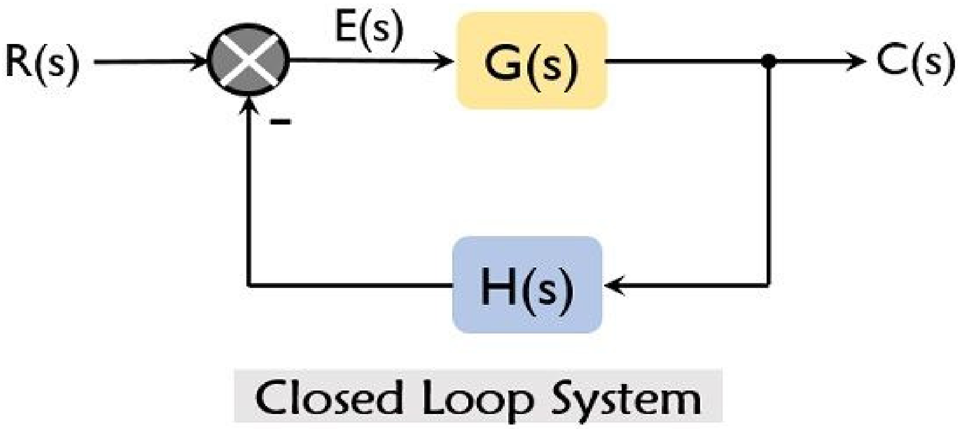
Figure 2.2: Illustrates the comprehensive block diagram depiction of a closed-loop control system.

Here as you can see how command input is delivered to a transducer. This is so because it is not always required that the provided input be accepted by the controller. Thus in such instances, the input cannot be immediately applied to the system. As it must be transformed from one form to another so that it may fulfill the job of reference input for the system. This is the reason input is originally applied towards the transducer so that it may be converted to a form accepted by the system according to the type of controller and process. When the controller creates the control signal as according input applied, therefore the needed action according to the created signal takes place within the system. This leads to the development of a certain output. But it is required to measure the created output in order to ascertain if it is the intended output or not. So, for this, a fraction of the obtained output is given to the input as well. This signal works as a feedback signal. This feedback signal, whenever compared against reference input, creates an error signal. This error signal is further supplied to the controller that creates a modified signal (proportional to error signal) which is simply a control input that leads the process to remove the error hence providing the desired output. The obtained output is known as the regulated output data and holds accuracy.

Transfer Function of Closed-Loop Control System

Transfer function displays the performance of the system because it is defined as that of the mathematics relation between both the input and output produced by the system. The gain of the system determines the ratio of output to input. Thus we may say the outcome of the system is the product of Fourier transform and input[7][8]–[11].

Consider the closed-loop system described below:



So, for the above-given system,

$$C(s) = E(s) \, . \, G(s)$$

$$E(s) = R(s) - H(s)C(s)$$

On substituting the value of E(s) in the 1st equation

$$C(s) = [R(s) - H(s)C(s)]. \, G(s)$$

$$C(s) = R(s) \, G(s) - H(s)C(s) \, G(s)$$

On transposing

$$R(s) \, G(s) = C(s) + H(s)C(s) \, G(s)$$

$$R(s) \, G(s) = C(s).[1 + G(s) \, H(s)]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{[1 + G(s) \, H(s)]}$$

This is the transfer function of a closed-loop system with negative feedback.

For a positive feedback system, it is given as:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{[1 - G(s) \, H(s)]}$$

For a unity feedback system (i.,e H(s) = 0) with a large amount of gain. The frequency response is unity.

Examples of Closed-Loop Control System

Almost all know that what a control system supervises or directs the functioning of a system so order to generate a specified output. In a closed-loop system, the intended output is attained by conducting a comparison between accomplished output and given input. And for this reason, a portion of the output is fed back to the controller in order to have the difference between the input and output value. This one is known as a feedback signal. Thus we may say the system whose functioning is regulated by its output is known as just a closed-loop control system. We have previously described a closed-loop control system in their previous post so to get a thorough notion of functioning you may refer the same. Here we will cover the instances along with the pros and downsides of closed-loop systems.

Examples of closed-loop control system

Here we shall describe the comprehensive functioning of an automated electric iron as well as a temperature control system.

Automatic Electric Iron

Consider the example of automated electric iron which functions as a closed-loop system. The image below illustrates the block diagram with main components inFigure 2.3:

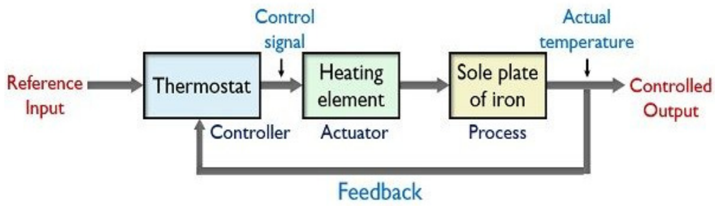


Figure 2.3: Illustrates circuit diagram the automatic electric iron.

An automated electric iron comprises of a thermostat that functions as a controller for the system, and a resistive heating component is provided that creates heat.

The sole-plate of both the iron instrument functions as a process of the whole system.

The fundamental functioning done by an automated electric iron is such that when the temperature of a sole-plate attains a specified value then the heating operation is terminated automatically. And when the temperature falls below a specific predetermined threshold then again heating begins within it.So, it is apparent that in this form of system the controlling relies on the output of the system.Initially, in electric iron, this same thermostat is given with a specified precise value which works as a reference input for the system.

When the input is delivered to the system, then perhaps the resistive heating element creates heat within the system. This leads to increasing up the temperature of an iron sole. Through a feedback mechanism, this output temperature is contrasted with the reference input of a thermostat. If the produced output displays smaller value than that of the reference input, then the differential temperature actuates its thermostat and thus turns on the heating element.

This resultantly creates a rise in the temperature of an iron sole.Once the temperature surpasses the reference value therefore the heating element automatically switches off. And after a given amount of time, the temperature begins to decline.However, the comparison still continues on and when the temperature falls below the precise value, the heating element immediately starts to increase the temperature of the sole.

In this approach the continual process inside of an electric iron actually took place.

Temperature Control System

Let us now consider another example of the control system for temperature that acts as a closed-loop system. The major objective offered by a temperature control system is to preserve a steady temperature of water. Generally, these systems are utilized to give an invariable temperature (hot) there at output.

The graphic below depicts the block diagram depiction of a closed-loop system inFigure 2.3:

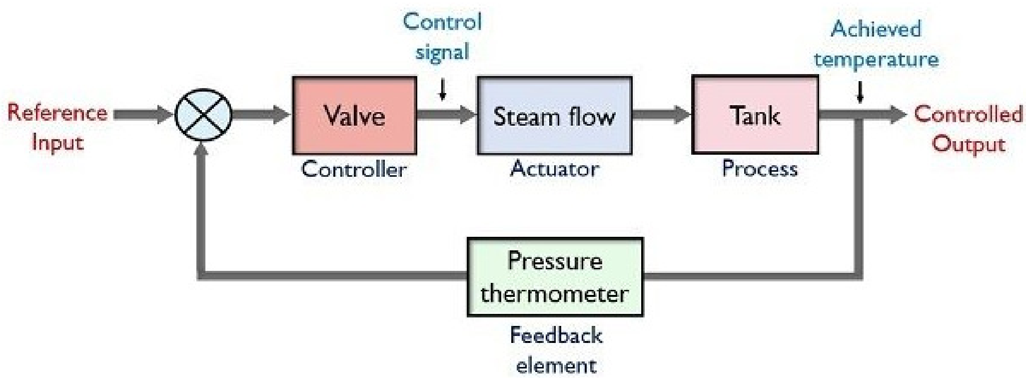


Figure 2.3: Illustrates the block diagram depiction of a closed-loop system.

Basically in such sort of systems water from an output arrive with a consistent flow rate. Also, internally produced steam from a valve is mixed with the water to obtain a set temperature of water. A pressure thermometer is employed within the system that functions as feedback. So, whenever a reference input is supplied to the system then the valve present provides a control signal that instructs the system to deliver the needed quantity of steam.

When the steam mixes with both the water flowing from the exit then the temperature of the liquid is monitored by that of the pressure thermometer and is contrasted with the reference input provided to the system.If the intended temperature (reference input) indicates equivalence with the produced temperature, therefore the control signal is created and the flow of steam is halted.But if any degree of fluctuation occurs between the two temperature readings then the controller produces the control signal about the level of temperature difference which is additionally corrected throughout the operation. In this approach, the continuous process within the system is taking place and a regulated level of temperatures is maintained.

Advantages

The closed-loop system is much more accurate than that of the open-loop system because of regulating via the output signal. These sorts of systems are less influenced by noise as well as other environmental perturbations. It gives a high-frequency range of operation as well as being more adaptable as compared towards the open-loop system inFigure 2.4.

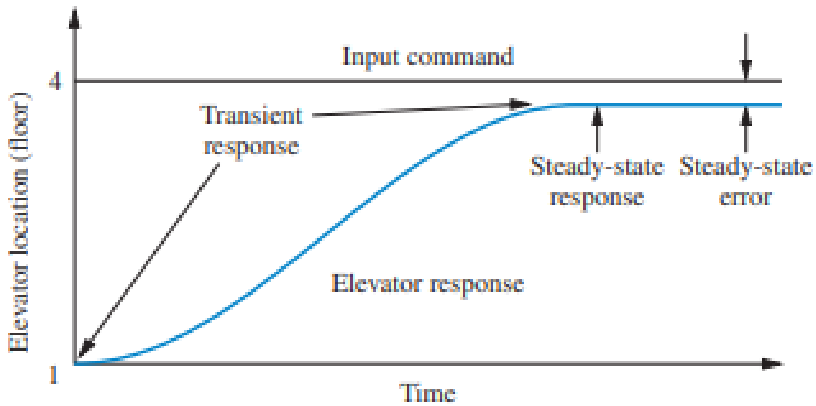


Figure 2.4: Illustrates the graphical representation of elevator versus time.

Disadvantages

The incorporation of the feedback components leads to the formation of complex structures.

Closed-loop systems really aren't economical.

The issue of instability in output is a critical component of the closed-loop system since the existence of feedback creates timely change in the system's output.

Applications

In our day to day existence, we come across different usage of closed-loop systems. From an air conditioner which manages to deliver the correct value of room temperature through making needed modifications to automated washing machines that impart the required degree of dryness to the fabric after washing. In a similar fashion from an automated toaster, water level controller, home heating system through dc motor speed control and missile launching system, etc. anything that is intended to deliver the desired output with precision is the closed-loop system.

Control systems for four key reasons:

Power amplification. Remote control Convenience of input form and Compensation for disturbances for example, a radar antenna, positioned by the low-power rotation of a knob there at input, needs a considerable amount of power for its output spin. A control system can create the desired power amplification, or power increase. Robots created using control system principles may compensate for human impairments. Control systems are also beneficial in isolated or risky situations. For example, a remote-controlled robot arm may be utilized to pick up stuff in a radioactive environment. A robot arm built to function in polluted settings. Control systems may also be utilized to improve convenience by modifying the shape of the input. For example, in a temperature management system, the input is a position on a thermostat. The output is heat. Thus, a handy location input gives a desirable thermal output. Another benefit of a control system is the capacity to correct for disruptions. Typically, we regulate basic variables as temperature in thermal systems, location and velocity in mechanical systems, and voltage, current, or frequency in electrical systems. The system must be able to give the right output even with a disruption. For example, imagine an antenna system that points inside a commanded direction. If wind pushes the antenna from its specified position, or if noise enters within, the system must be able to detect the disruption and rectify the antenna's position.

A History of Control Systems

Feedback control mechanisms are ancient than mankind. Numerous biological regulatory mechanisms were established into the early inhabitants of our planet. Let us now look at a short history of human-designed control systems.

Liquid-Level Control

The Greeks started building feedback systems circa 300 B.C. A water clock designed by Ktesibios functioned by having water drip into a measuring container at a steady pace. The amount of water in the measuring jug might be utilized to determine time. For rainwater to trickle at a consistent pace, the supply tank had to be maintained at a constant level. This was achieved utilizing a float valve similar towards the water-level control in today's flush toilets. Soon after Ktesibios, the principle of liquid-level control was applied to an oil lamp by Philon of Byzantium. The light consists of two oil canisters oriented vertically. The bottom pan was open from the top and provided the fuel source for the blaze. The lidded top bowl was the fuel storage for the pan below. The containers were coupled by two capillary tubes and then another tube, termed a vertical riser, which has been placed into the oil in the bottom pan slightly below the surface. As the oil burnt, the base of the vertical riser was exposed to the air, which drove oil in the reservoir above to flow down the capillary tubes and into the pan. The flow of fuel from the higher reservoir to the pan halted when the former oil level in the pan was restored, thereby keeping the air from entering through vertical riser. Hence, the method maintained the liquid level inside the bottom container constant.

Steam Pressure and Temperature Controls

Regulation of steam pressure started about 1681 with Denis Papin's creation of the safety valve. The notion was subsequently improved on by weighing the valve top. If the upward pressure from the boiler surpassed the weight, steam was released, as well as the pressure reduced. If it did not surpass the weight, the valve didn't open, and the pressure within the boiler grew. Thus, the weight here on valve top controlled the internal pressure of a boiler.

Also in the seventeenth century, Cornelis Drebbel in Holland created a totally mechanical temperature control device for hatching eggs. The gadget employed a vial of alcohol and mercury with a floater placed in it. The floater was attached to a damper that regulated a flame. A part of the vial was placed into the incubators to feel the heat created by the fire. As the heat grew, the alcohol and mercury expanded, lifting the floater, shutting the damper, and decreasing the flame. Lower temperature led the float to sink, opening the damper thus intensifying the flame.

Speed Control

In 1745, speed control was used to a windmill by Edmund Lee. Increasing winds pitched the blades further back, so that less surface was accessible. As the wind lessened, more blade surface was accessible. William Cubitt built on the concept in 1809 by separating the windmill sail into moveable louvers. Also during the eighteenth century, James Watt devised the flyball speed governor to regulate the speed of steam engines. In this gadget, two spinning flyballs ascend as rotational speed rises. A steam valve attached to the flyball mechanism shuts with the rising flyballs and opens with both the descending flyballs, thereby controlling the speed.

Introduction Stability, Stabilization, and Steering

Control systems theory as we know it now started to coalesce in the later part of the nineteenth couple of centuries. In 1868, James Clerk Maxwell presented the stability criteria for a third-order system that relies on the coefficients of the differential equation. In 1874, Edward John Routh, following a proposal from William Kingdon Clifford that was overlooked earlier by Maxwell, was capable of expanding the stability criteria to fifth-order systems. In 1877, the subject for the Adams Prize was “The Criterion of Dynamical Stability.” In response, Routh submitted a paper entitled A Treatise on the Stability of such a Given State of Motion and received the award. This work includes what is now known as the Routh-Hurwitz criteria for stability, which we shall discus. Alexandr Michailovich Lyapunov also contributed to the creation and formulation of today’s theories and practice of control system instability. A student of P. L. Chebyshev at the University of St. Petersburg in Russia, Lyapunov expanded the work of Routh to nonlinear systems in his 1892 PhD thesis, entitled The General Problem of Stability of Motion. During the second part of the 1800s, the development of control systems centered on the steering and stabilization of ships. In 1874, Henry Bessemer, using a gyro to measure a ship’s motion and employing power supplied by the ship’s hydraulic system, relocated the ship’s saloon to keep it steady (whether this had an impact to the customers is dubious) (whether this made a difference to the patrons is doubtful). Other attempts were made to steady platforms for cannons as well as to stable whole ships, employing pendulums to detect the motion. Twentieth-Century Developments It was not until the early 1900s that autonomous steering of ships was established. In 1922, the Sperry Gyroscope Company implemented an automated steering system that employed the concepts of compensation and adaptive control to increase performance. However, most of the basic theory utilized today to enhance the efficiency of automated control systems is ascribed to Nicholas Minorsky, a Russian born around 1885. It was his theoretical work applied to the autonomous steering of ships that resulted to what we call today proportional-plus-integral-plus-derivative (PID), or three-mode, controllers, which we shall cover.

Inside the late 1920s and early 1930s, H. W. Bode and H. Nyquist of Bell Telephone Laboratories pioneered the analysis of feedback amplifiers. These contributions developed into sinusoidal frequency analysis and design methodologies now utilized for feedback control system, and are discussed. In 1948, Walter R. Evans, working in the aviation industry, invented a graphical approach to draw the roots of a characteristic equation of a feedback system wherein parameters fluctuated across a given range of values. This method, now known as the root locus, takes its place alongside the work of Bode and Nyquist in establishing the cornerstone of linear control systems analysis and development theory. Contemporary Applications Today, control systems find significant use in the guidance, navigation, and control of missiles and spacecraft, as well as aircraft and ships at sea. For example, contemporary ships utilize a mix of electrical, mechanical, and hydraulic components to create rudder orders in response to desired heading commands. The rudder orders, in turn, result in a rudder angle that directs the ship. We find control systems across the process control business, managing liquid levels in containers, chemical concentrations in vats, as well as the thickness of manufactured material. For example, imagine a thickness control system for such a steel plate finishing factory. Steel enters the finishing mill and goes through rollers. In the finishing machine, X-rays measure the actual thickness and compare it to the target thickness. Any discrepancy is controlled by a screw-down position adjustment that alters the roll gap at the crushers through which the steel travels. This honge in roll gap determines the thickness. Modern improvements have seen extensive usage of the digital computer as component of control systems. For example, computers in control systems are really for industrial robots, spaceships, and the process control sector. It is impossible to conceive a contemporary control system that does not involve a digital computer.

Although recently decommissioned, the space shuttle offers an outstanding illustration of the employment of control systems since it featured several control systems managed by an onboard computer on the a time-shared basis. Without control systems, it would be difficult to maneuver the shuttle to and from earth's orbit or to alter the orbit itself and maintain life on board. Navigation routines incorporated into the shuttle's computers utilised data from the shuttle's hardware and estimate vehicle location and velocity. This information was sent to the guidance equations that calculated orders for the shuttle's flight control systems, effectively directed the vehicle. In space, the flight control system gimbaled (rotated) three orbital maneuvering system (OMS) engines together into position that produced thrust in the specified direction to steer the ship. Within the earth's atmosphere, the shuttle was directed by orders issued from the flight control system towards the aerosurfaces, such as the elevons. Within this huge control system represented by navigation, navigation, and control were various subsystems to govern the vehicle's activities. For example, the elevons needed a control system to guarantee that their position was really that which was intended, as disturbances like as wind may spin the elevons away from the prescribed position. Similarly, in spacecraft, the gimbaling of the orbital maneuvering engines requires a similar control system to guarantee that the rotating engine can complete its duty with speed and precision. Control mechanisms were also employed to control and steady the spacecraft during its fall from orbit. Numerous tiny jets that constitute the response control system (RCS) were utilized originally in the exoatmosphere, where even the aerosurfaces are useless. Control was given to the aerosurfaces as the orbiter fell into the atmosphere. Inside the shuttle, several control systems were necessary for electricity and life support. For example, the orbiter contained three fuel-cell power plants that turned hydrogen and oxygen (reactants) into energy and water for use by the crew. The fuel cells featured the use of control systems to manage temperature and pressure. The reactant tanks were held at constant pressure while the amount of reactant lowers. Sensors in the tanks transmitted signals to the control systems to switch heaters on or off to maintain the tank pressure constant (Rockwell International, 1984). Control systems are not confined to science and industry. For example, a home heating system is a basic control system consisting of a thermostat incorporating a bimetallic substance that expands or contracts upon changing temperature. This expansion or contraction moves a vial of mercury that functions as a switch, turning the heater on or off. The amount of movement or contraction needed to move the mercury switch is dictated by the temperature setting. Home entertainment systems also include built-in control systems. For example, in an optical disk recording system small pits representing the information are burnt into the disc by a laser during the data collection process. In playback, a reflected laser beam focused on the pits changes intensity. The light intensity variations are translated to an electrical signal and processed as sound or image. A control system maintains the laser pointer positioned on the pits, which are cut as concentric circles. There are innumerable additional instances of control systems, from the commonplace to the remarkable. As you begin your study in control information systems, users will become more aware of the huge array of applications.

System Configurations

In this part, we cover two main configurations of control systems: open loop and closed loop. We may consider these combinations to constitute the internal architecture of an overall system depicted in Figure 2.5 and 2.6. Finally, we explain how a digital computer becomes part of a control scheme configuration.

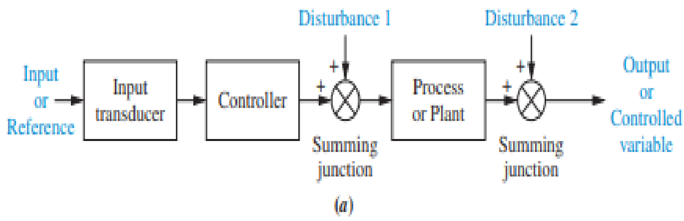


Figure 2.5: Illustrates the configuration of open loop system.

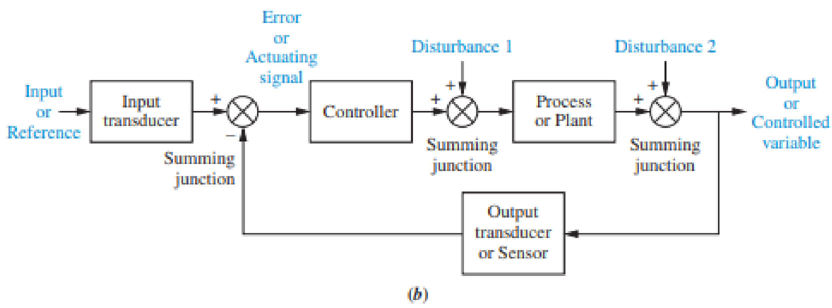


Figure 2.6: Illustrates the configuration of open closed system.

Computer-Controlled Systems

In many contemporary systems, the controller (or compensator) is a digital computer. The benefit of utilizing a computer is that several loops may be managed or compensated by the same computer via time sharing. Furthermore, any modifications of the compensator parameters necessary to give a desirable response may be accomplished by changes in software rather than hardware. The computer may also perform supervisory activities, such as scheduling various essential programs. For example, the space shuttle main engine (SSME) controller, which comprised two digital computers, alone regulated multiple engine operations. It monitored engine sensors that gave pressures, temperatures, flow velocity, turbo-pump speed, valve positions, and engine servo dependent variable positions. The controller additionally offered closed-loop control of propulsion and propellant mixture proportion, sensor excitation, valve actuators, spark igniters, as well as additional functions.

Transient Response

Transient reaction is significant. In the instance of an elevator, a delayed transitory reaction makes passengers antsy, but an abnormally quick response makes them uncomfortable. If the elevator oscillates around the arrival floor for more than a second, an uncomfortable sensation might develop. Transient response is also significant for structural reasons: Too quick a transitory reaction might cause irreversible physical harm. In a computer, transient response adds to the time needed to read from or write to the computer's disk storage. Because reading and writing cannot take place again until head stops, the speed of both the read/write head's movement through one track upon that disk to another effects the overall speed of the computer. In this book, they provide quantitative concepts for transitory reaction. We then assess the system for its present transient response. Finally, we alter parameters or design components to give a desirable transient response—our initial analytical and design aim. Steady-State Response Another study and design aim focuses on the steady-state response. As we've already seen, this response mimics the input and is generally what that remains after transients have declined to zero. For example, this answer may be an elevator halted at the fourth level or the head of a storage device finally stopped there at proper track. We are worried about the correctness of the steady-state answer. An elevator must be level sufficient with the floor therefore for passengers to escape, and a read/write head not positioned and over required track results in computer failures. An antenna monitoring a satellite must maintain the spacecraft substantially inside its beam width in order to avoid losing track.

In this book chapter we define steady-state errors statistically, assess a system's steady-state error, and then plan corrective action to lower the steady-state error—our second analysis and design aim. Computer hard disk drive, displaying disks and read/write head 1.4 Analysis and Development Objectives 9 Stability Discussion of transient responsiveness and steady-state error is meaningless if the system does not have consistency. In order to understand stability, we start with the notion that the overall reaction of a system is the sum of the natural response and the forced response. Whenever you researched linear differential equations, students presumably referred to these replies as the homogeneous and the specific solutions, respectively. Natural reaction explains the way the system dumps or gains energy. The shape or character of this reaction is reliant entirely on the system, not the input. On the contrary hand, the shape or character of the forced reaction is depending on the input. Thus, given a linear system, we may write Total reaction ˆ Natural reaction Forced reaction … 1.1† 2 for a control system to be functional, the natural response must (1) gradually approach zero, thereby leaving just the forced reaction, or (2) oscillate. In certain systems, however, the natural response rises without limit rather than shrink to zero or fluctuate. Eventually, the natural reaction is so much stronger than the artificial response that the system is no longer controlled. This phenomenon, termed instability, might lead to self-destruction of a physical device if limit controls are not part of the design. For example, the escalator would shut down through the floor or exit through the ceiling; an aircraft would have to go into an uncontrollable roll; or an antenna directly ordered to point to a target will indeed rotate, line up with the target, but then start to oscillate about just the target with growing oscillations and vastly increased velocity again until motor or amplifiers reached their production limits or up until the antenna has been damaged structurally.

A time plot of an undamped system would show a transitory response that increases without limit and without any sign of a steady-state response. Control systems must always be built to be stable. That is, their natural reaction must decrease to zero as time approaches infinity or oscillate. In many technologies the transient reaction you observe on a temporal response plot may be directly connected to the natural response. Thus, if the natural reaction decays to zero as the time approaches infinity, the transitory response will likewise fade out, leaving just the forced response. If the system is stable, the suitable transient response and steady-state error characteristics may be developed. Stability is your third analysis and design aim. Other Considerations The three major goals of control system analysis and design have previously been identified. However, other essential variables must be taken into consideration. For example, variables impacting hardware choices, such as motor size to satisfy power needs and choice of sensors for precision, must be addressed early in the design. Finances are another factor. Control system designers cannot produce designs without considering potential economic effect. Such concerns as budget allocations and competitive rates must influence the engineer. For example, if your product is a one-of-a-kind kind, then may be able to construct a design that incorporates more costly components without materially raising overall cost. However, when your design will be utilized for numerous copies, modest increases in cost per copy might translate into hundreds of additional dollars for your organization to present during contract bidding and to spend before sales. Another factor is strong design.

System characteristics presumed constant throughout the design for transient responsiveness, steady-state errors, and durability vary over time when the real system is produced. Thus, the performance of the system also varies over time and is unlikely to be consistent with your design. Unfortunately, the connection between parameter modifications and their effects on performance is not linear. In certain circumstances, even in the same system, changes changing parameter values might lead to slight or major variations in performance, dependent on the system's normal operating point and the kind of design utilized. Thus, the engineer aims to construct a robust design in order that the system will not be susceptible to parameter changes. The Design Process In this part, we construct an organized procedure for the design of feedback control systems that is to be followed as we go through the remainder of the book. It illustrates the outlined method as well as the chapters where the phases are covered. The antenna azimuth position control system presented in the previous section is illustrative of control systems that must be examined and constructed. Intrinsic is feedback and communication throughout each step. For example, if testing (Step 6) indicates that criteria have not been satisfied, the system must be modified and retested. Sometimes criteria are contradictory as well as the design cannot be reached. In these circumstances, the requirements are required to be established and the design process redone. Let us now elaborate on each block

Transform Requirements

Into a Physical System We begin by translating the requirements into a physical system. For example, inside the antenna azimuth position control system, this same requirements would express the desire to position. The antenna from a faraway point and explain such factors as weight and physical dimensions. Using the requirements, design criteria, such as desirable transient responsiveness and steady-state accuracy, were derived in Figure 2.6. Perhaps an overarching idea, would result.

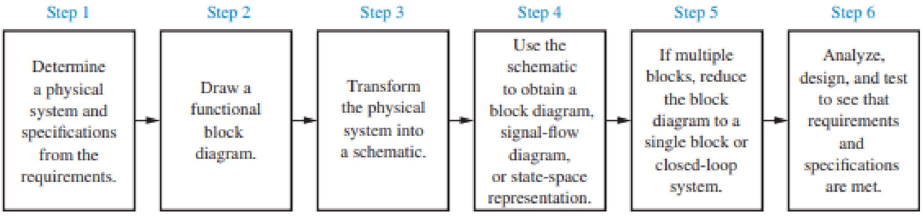


Figure 2.6: Illustrates the block diagram of transform requirement.

The designer now turns a qualitative description of the system into a functional block diagram that explains the component pieces of the system (that is, function and/or hardware) and depicts their connectivity.

Create a Schematic As we have seen, position control systems comprise of electrical, mechanical, and electromechanical components. After developing the description of a physical system, professional control systems engineer translates the actual system into a schematic diagram. The control system designer may begin with both the physical description, to create a schematic. The engineer must make estimates about the system and overlook some processes, or else the schematic would be bulky, making it impossible to derive a suitable mathematical model during the following step of the analysis and design sequence. The designer begins with a basic schematic representation and, at succeeding steps of the analysis and design process, evaluates the assumptions made about the physical system via analysis but instead computer simulation. If the design is too simplistic and does not fully account for observable behaviors, the control systems engineer introduces phenomena towards the schematic that were previously thought insignificant. A schematic design for the antenna azimuth position control system. When we draw the potentiometers, then establish our first implicit assumption by omitting their friction or inertia. These mechanical features provide a dynamic, rather than an immediate, reaction in the output voltage. We presume that these mechanical effects are insignificant and that the voltage across a potentiometer changes immediately as the potentiometer shaft spins. A differential amplifier as well as a power amplifier are utilized as the controller to provide gain and output amplification, correspondingly, to drive the motor. Again, humans assume that perhaps the dynamics of an amplifiers are quick relative to the reaction time of the motor; consequently, we represent them as a pure gain, K. A dc motor and comparable load create the output angular displacement.

The velocity of the motor is proportionate to the voltage delivered to the motor's armature winding. Both inductance and resistance are elements of the armature circuit. In illustrating merely the armature resistance, they assume the influence of the armature inductance is insignificant for a dc motor. The designer makes more assumptions about the load. The load consists of a spinning mass and bearing friction. Thus, the model comprises of inertia and viscous damping whereby resistive torque increases increasing speed, such as an automobile's suspension system or a screen door damper. The judgments taken in constructing the schematic originate from understanding of the physical system, the physical rules guiding the system's behavior, including practical experience. These judgments are not simple; nevertheless, as you acquire additional design expertise, you will obtain the knowledge necessary for this challenging endeavor.

The Design Process 15 Step:

Develop a Mathematical Model (Block Diagram) (Block Diagram) that once schematic is constructed, the designer employs physical rules, such as Kirchhoff's laws with electrical networks but also Newton's law for mechanical devices, combined with simple assumptions, to represent the system mathematically. These laws are

1. Kirchhoff's Voltage Law
2. Kirchhoff's Current Law
3. Newton's Law

Kirchhoff's and Newton's laws lead towards mathematical models that explain the link between the inputs and outputs of dynamic systems. One such model is the nonlinear, time-invariant differential equation, Equation.

$$\frac{d^m c(t)}{dt^n} + d_{n-1}\frac{d^{m-1}c(t)}{dt^{n-1}} + \cdots + d_0 c(t) = b_m\frac{d^m r(t)}{dt^m} + b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

Many systems may be roughly characterized by this equation, which ties the output, c(t), to the input, r(t), by means of the system parameters, ai and bj. We presume the reader is acquainted with differential equations. Problems and a bibliography are supplied at the conclusion of the chapter for you to review this topic. Simplifying assumptions made inside the process of developing a mathematical model frequently leads to a low-order form. Without the assumptions the system model might be of high order or characterized using nonlinear, time-varying, or partial differential equations. These equations complicated the design process and diminish the designer's understanding. Of course, all assumptions must be validated and all simplifications justified by analysis or testing. If the assumptions for simplification can indeed be justified, then perhaps the model cannot be simplified.

In complement to the differential equation, the transfer function is another approach of mathematically representing a system. The model is constructed from the linear, time-invariant differential equation to use what we call the Laplace transform. Although the transfer function may be utilized exclusively for linear systems, it offers more comprehensible information than that of the differential equation. Users will be able to adjust system parameters and instantly perceive the influence of these changes on the system reaction. The transfer function is also helpful in representing the interconnectedness of subsystems by generating a block diagram similar to but including a mathematical function within each block. Still another paradigm is the state-space representation. One benefit of state-space approaches is that they may also be utilized for systems that cannot be represented by linear difference equations. Further, state-space approaches are utilized to represent systems for simulation just on digital computer. Basically, this form converts an nth-order differential problem into n simultaneous first-order differential equations. Let this explanation enough for now; we discuss this strategy in greater depth.Finally, we should highlight that to construct the mathematical model for a system, we need information of the parameter values, including such equivalent resistance, conductance, mass, and damping, which is frequently not simple to get. Analysis, measurements, or specifications from vendors are sources that the control engineering team may utilize to gather the parameters.

Reduce the Block Diagram Subsystem models are combined to construct block diagrams of bigger systems, where each block has a mathematical explanation. Notice that so many signals, such as proportional voltages and error, are intrinsic to the system. There are additionally two signals—angular input but also angular output—that are external to the system. In order to assess system response in this example, we need to compress this enormous system's block diagram to something like a single block with such a mathematical description that depicts the system from its input to its outputs, as illustrated. Once the block diagram is simplified, we are ready to study and design the system. Analyze and Design the next step of the process, after block diagram reduction, is planning and analysis. If you are interested just in the performance of a specific subsystem, however may skip the block diagram reduction and proceed right into analysis and design. In this step, the engineer studies the system to determine whether the response specifications and performance criteria may be fulfilled by simple tweaks of system characteristics. If requirements cannot be reached, the designer then creates extra hardware in order to achieve a desired performance. Test input signals are employed, both analytically and during testing, to validate the design. It is not necessary practicable nor informative to pick sophisticated input signals to study a system's performance. Thus, the engineer normally picks conventional test inputs. These inputs are impulses, steps, ramps, parabolas, and sinusoids, as illustrated in Table 1.1. An impulse is infinite at t ˆ 0 and zero everywhere. The area underneath the unit impulse is 1. An approximation of this sort of waveform is used to introduce initial energy into a system such that the response owing to that initial energy is simply the transitory response of a system. From this answer the designer may develop a system's mathematical model from this information. A step input represents a constant instruction, such as location, velocity, or acceleration. Typically, the step input instruction is of the same form as the output. For example, if the system's output is positioning, as it is for the antenna azimuth position control system, the step input represents a desired position, and the output reflects the actual position. If indeed the system's output is velocity, as example the spindle speed for the video disc player, the step input indicates a constant intended speed, while the output reflects the actual speed.

The designer chooses step inputs since both the transient reaction and the steady-state response are readily apparent and can be analyzed. The ramp input provides a linearly ascending message. For example, if the system's output is location, the input ramp depicts a linearly rising position, such as that encountered while following a satellite traveling across the sky at constant speed. If the system's output is speed, the input ramp reflects a linearly rising velocity. The response to the input ramp test signal offers further information about just the steady-state error. The same concept may be generalized to parabolic inputs, which also are likewise utilized to assess a system's steady-state error. Sinusoidal inputs may also be used to examine a physical system and arrive at the mathematical model.

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CHAPTER 3

CONTROL SYSTEMS ENGINEER

Dr. Balaji K.A, Assistant Professor, Selection Grade
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- balaji.ka@presidencyuniversity.in

Control systems engineering is an intriguing area in which to use your engineering abilities, since it spans across multiple disciplines and various roles within those fields. The control engineer may be found at the highest level of big projects, participating at the conceptual phase in identifying or executing overall system needs. These requirements comprise overall system performance parameters, subsystem functions, and the interconnection of these functions, including interface requirements, hardware and software design, and test plans and procedures. Many engineers are involved in just one field, such as circuit design or application design. However, as a control systems engineer, anyone may find yourself working in a wide arena and dealing with experts from different fields of engineering and the sciences. For example, when you are working on a biological system, you will need to engage with colleagues inside the biological sciences, mechanical engineering, electrical engineering, and computer engineering, not to mention physics and mathematics. You will be collaborating with these engineers at all phases of project development from idea through design and, eventually, testing. There at design level, the control systems engineer might be conducting hardware selection, design, and interface, including whole subsystem design to fulfill stated criteria. The control engineer might be dealing with sensors and motors in addition to electrical, pneumatic, and hydraulic circuits. The space shuttle is another illustration of the variety demanded of the systems engineer[1]–[5].

We demonstrated that the space shuttle's management systems crossed across several disciplines of science: orbital mechanics and propulsion, aerodynamic electrical engineering, and mechanical engineering. Whether or whether you work in the space program, as a control systems engineer visitors will use broad-based expertise to the solution of engineering control challenges. You will have the chance to develop their engineering horizons beyond your academic program. You are now aware of future prospects. But for now, what benefits does this course give to a student studying control systems (other than the fact that you need it to graduate)? Engineering programs tend to stress bottom-up design. That is, you start with the components, build circuits, and then construct a product. In top-down design, a high-level image of the requirements is first developed; then the functions and hardware necessary to implement the system are defined. You will be able to adopt a top-down systems approach as a consequence of this training. A key reason for not teaching top-down design across the curriculum is the high degree of mathematics initially necessary for the systems approach. For example, control systems theory, which needs differential equations, could not be taught as a lower-division subject. However, when advancing through bottom-up design classes, it is difficult to grasp how such design fits logically into the wider picture of the product development cycle. After finishing this control systems course, you will be able to step back and see how your prior studies fit into the big picture. Your amplifier course or vibrations course will take on new significance as you begin to comprehend the role design work plays as part of product development. For example, as engineers, we seek to explain the physical world mathematically so that we may design systems that will benefit mankind. You will discover that you have actually learned, via your prior courses, the capacity to model physical systems numerically, but at the time you may not have recognized where within the product development cycle the modeling fits. Understanding control systems helps students from all areas of engineering to communicate a common language and build appreciate and practical knowledge of the other branches. You will discover that there actually is not much variation among the fields of engineering as far as the aims and applications were concerned.

Modeling in the Frequency Domain

A differential equation may explain the connection between the input and output of a system. The form of a differential equation as well as its coefficients represent a formulation or description of the system in Figure 3.1. Although the differential equation ties the system towards its input and output, it is not a suitable representation from a system standpoint. Looking a generic, nth-order, linear, time-invariant differential equation, you find that the system parameters, which are the coefficients, exist throughout the equation. In addition, the output, c (t), as well as the input, r(t), both occur throughout the equation. We would like a mathematical formulation such as that provided, where the input, output, and system were distinct and independent portions. Also, we would want to depict easily the interconnectedness of many subsystems. For example, we would want to describe cascaded interconnections, as illustrated, in which a mathematical function, termed a transfer function, is within each block, and chunk functions may easily be coupled to give for simplicity of analysis and design. This convenience cannot be reached using the differential equation.

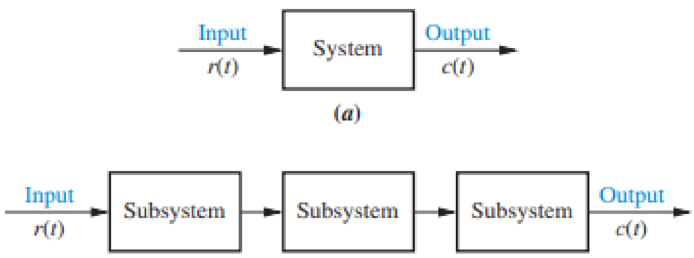


Figure 3.1: Illustrates the connection between the input and output of a system.

Laplace Transform Review

A system described by a differential equation is difficult to depict as a block diagram. Thus, we now establish the framework for the Transfer function, with with which we can represent both input, output, and system as independent entities. Further, their interaction will be simple algebraic. Let us first describe the Laplace transform and afterwards illustrate how it simplifies the representations of physical systems. The Laplace transform was defined as,

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

where s ˆ σ jω, a complex variable. Thus, knowing f(t) and assuming that the integral in Eq. exists, we may derive a function, F(s), that is termed the Laplace transform of f(t) (t). The notation for the lower limit suggests that even if f(t) is discontinuous at t ˆ 0, one may start our integration prior to the discontinuity thus long as the integral converges. Thus, we may determine the Transformation function of impulse functions. This trait has particular benefits when implementing the Laplace transform towards the solution of differential equations whenever the starting conditions are discontinuous at t ˆ 0. Using differential equations, researchers have had to solve for the starting circumstances after the discontinuity understanding the beginning conditions before even the discontinuity. To use the Laplace transform humans need only know the beginning circumstances before the discontinuity. The inverse Laplace transform, that enables us to calculate f(t) given F(s) (s), is

$$\mathscr{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}\,ds = f(t)u(t)$$

where

$$u(t) = 1 \quad t > 0$$
$$= 0 \quad t < 0$$

The Transfer Function

In the last section we defined the Laplace transform as well as its inverse. We discussed the notion of the partial-fraction expansion but also applied the principles to the differential equations we were solving. We are now ready to develop the system representation depicted in Figure 2.1 by creating a feasible specification for a function which algebraically ties a system's output to its input. This function will enable segregation of the input, system, and output into three discrete and distinct portions, unlike the differential equation[5]–[8]. The function will also enable us to algebraically integrate computational models of subsystem to create a comprehensive system description. Let us commence by formulating a generic nth-order, linear, time-invariant differential equation:

$$a_n\frac{d^n c(t)}{dt^n} + a_{n-1}\frac{d^{n-1}c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m\frac{d^m r(t)}{dt^m} + b_{m-1}\frac{d^{m-1}r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

where $c(t)$ is the output, $r(t)$ is the input, and the a_i's, b_i's, and the form of the differential equation represent the system. Taking the Laplace transform of both sides,

$$a_n s^n C(s) + a_{n-1}s^{n-1}C(s) + \cdots + a_0 C(s) + \text{initial condition terms involving } c(t)$$
$$= b_m s^m R(s) + b_{m-1}s^{m-1}R(s) + \cdots + b_0 R(s) + \text{initial condition}$$

$$(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0)C(s) = (b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0)R(s)$$

Now form the ratio of the output transform, $C(s)$, divided by the input transform,

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0)}$$

Electrical Network Transfer Functions

In this part, we explicitly apply the transfer function to the mathematical modeling of electric circuits comprising passive networks and operational amplifier circuits. Following subsections discuss mechanical and electromechanical components. Equivalent circuits for the electric networks that we deal with initially consist of three passive linear components: resistors, capacitors, plus inductors. The components as well as the relationships involving voltage and current and between voltages but instead charge under zero starting circumstances. We now integrate electrical components into circuits, decide upon that input and output, and determine the transfer function. Our guiding concepts are Kirchhoff's laws. We add voltages around loops or sum currents at nodes, dependent on which approach entails the least effort in algebraic manipulation, and afterwards equal the result to zero. From these correlations we can build the differential equations again for circuit. Then we may use the Laplace transforms of differential equations and ultimately solve for such transfer function[9], [10].

Simple Circuits through Mesh Analysis

Transfer functions may be constructed using Kirchhoff's voltage law and summing voltages across loops or meshes.

Term this technique loop or mesh analysis and explain it through the following example. Let us now build a method for simplifying the answer for future problems. First, consider the Laplace transform of something like the equations inside the voltage-current presuming zero conditions.

For the capacitor,

$$V(s) = \frac{1}{Cs} I(s)$$

For the resistor,

$$V(s) = RI(s)$$

For the inductor,

$$V(s) = LsI(s)$$

Now define the following transfer function:

$$\frac{V(s)}{I(s)} = Z(s)$$

Notice that this function is comparable to the concept of resistance, that is, the ratio of voltage to current. However, unlike resistance, this function is appropriate to capacitors and inductors and conveys information about the dynamic behavior of the component, as it forms an analogous differential equation. They name this specific transfer function impedance. The impedance for every one of the electrical elements was presented. Let us now explain how the idea of impedance simplifies the equation for the transfer function. The Laplace transform of Eq, considering zero starting conditions, is:

$$\left(Ls + R + \frac{1}{Cs}\right) I(s) = V(s)$$

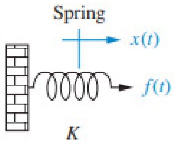
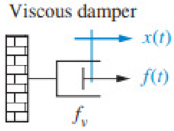
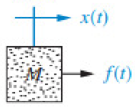
Translational Mechanical System Transfer Functions

We have demonstrated that electrical networks may be described by a transfer function, G(s), that algebraically connects the Laplace transform of the output toward the Laplace transform of both the input. Now we shall apply the same process to mechanical systems. In this section we focus on translational mechanical systems. In the following part we apply the principles to rotating mechanical systems. Notice that the finished result, depicted in Figure 2.2, would be mathematically indistinguishable from an electricity systems. Therefore, an electrical network may be interfaced to a mechanical system through cascading their difference equation, provided that one system is not burdened by the other. 6 Mechanical systems parallel electrical networks to such a degree that there are parallels between systems and equipment and variables. Mechanical systems, including electrical networks, comprise three passive, linear components. Two of these, the spring and the mass, are energy-storage components;

one of them, the viscous damper, releases energy. The two energy-storage components are equivalent to the two electrical energy-storage components, the inductor and capacitor. The energy dissipator is equivalent to electrical resistance. In the table, K, f v, and M are named spring constant, coefficient of viscous friction, as well as mass, respectively.

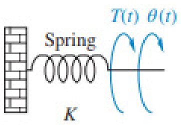
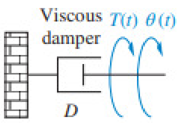
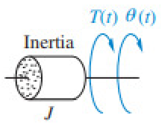
We observe that the spring is equivalent to the capacitor, the viscous damper is comparable to the resistor, and the masses is comparable to the inductor. Thus, summing forces expressed in terms of velocity is equivalent to summing voltages written in terms of current, and the resultant mechanical differential equations are analogous to meshes equations. If the forces are shown in terms of displacement, the resultant mechanical equations resemble, but they are not comparable to, the mesh equations. We, however, shall utilize this model for mechanical systems so that we may write equations directly in terms of displacement. Another connection may be established by comparing the force-velocity column of to the current-voltage column of in reverse order.

Here that the analogy is between force and current and between velocity and voltage. Also, the spring is equivalent to the inductor, the viscous damper has analogous to the resistor, and the mass is analogous to the capacitor. Thus, summing forces expressed in terms of velocity is equivalent to summing currents written throughout terms of voltage and the resultant mechanical differential equations are akin to nodal equations. We shall analyze these comparisons in greater depth. We are now able to identify transfer functions with translational mechanical systems. The mechanical system needs only one differential equation, termed the equation of motion, to explain it. We will begin simply assuming a positive motion, such as for example, towards the right. This presumed positive direction of motion is comparable to assuming a current direction inside an electrical loop. Using our assumed orientation of positive motion, we first build a free-body diagram, putting on the body any forces that operate on the body either in the direction of movement or opposing to it. Next we utilize Newton's law to build a differential motion equation by accumulating the forces and putting the total equal to zero. Finally, assuming zero starting conditions, we take the Laplace transform of a differential equation, differentiate the variables, and arrive just at transfer function. An example follows:

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Rotational Mechanical System Transfer Functions

Having studied electrical and translational mechanical systems, we now go on to explore rotating mechanical systems. Rotational mechanical systems are treated the same manner as translational mechanical systems, except that torque substitutes force and angular displacement substitutes translational displacement. The mechanical components for rotating systems are identical to those for translational systems, however the components undergo rotation instead of translation. The components together with the correlations between torque and angular velocity, in addition to angular displacement. Notice that the symbols for both the components appear the same as translational symbols, yet they are undergoing rotation but not translation. Also observe that the phrase linked with the mass is substituted with inertia. The quantities of K, D, and J are termed spring constant, coefficient of viscous friction, and moment of inertia, correspondingly. The inductances of the mechanical components are likewise reported in the final column. The values may be determined by applying the Laplace transform, presuming zero starting conditions, of the torque-angular deformation column. The idea of degrees of freedom transfers over to rotational systems, with the exception that we test a point of motion by rotating something while keeping still all other points of motion. The amount of points of motion which can be rotated whilst all others are kept stationary equals the amount of equations of motion necessary to describe the system. Writing the equations of motion for rotating systems is identical to writing those for translational systems; the main difference is that the free-body diagram comprises of torques rather than forces. They derive these torques via superposition. First, we rotate a body while keeping all other points constant and display on its free-body diagram all torques owing to the body's own motion. Then, keeping the body steady, we rotate neighboring points of motion one by one time and add the torques owing to the adjacent motion towards the free-body diagram. The process is repeated by each point of motion. For every free-body diagram, these torques have been summed as well as set to zero in order to form the motion equations.

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Transfer Functions for Systems with Gears

Now that we are able to calculate the transfer function for rotating systems, we recognize that spinning systems, particularly those powered by motors, are seldom observed without related gear trains driving the load. This section discusses this crucial subject. Gears offer mechanical advantage to rotating systems. Anybody who has ridden a 10-speed bicycle understands the impact of gearing. Going upward, you shift to offer more torque and less speed. On the straightaway, they shift to achieve greater speed and less torque. Thus, gears enable developers to match the driving system and the load—a trade-off between speed and torque. For many applications, gears show backlash, which happens due of the slack fit between two meshed gears. The driving gear spins over a minor angle before making contact with the meshed gear. The consequence is that the angular revolution of the output gear does not occur until a modest angular rotation of the input gear has happened. In this section, we idealize the behaviour of gears and assume that there is no backlash. The linearized relationship between two gears is represented in Figure 2.27. Thus,

$$r_1\theta_1 = r_2\theta_2$$

$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

Since the ratio of the number of teeth around the circle is in the same proportion as that of the ratio of the radii. They find that the ratio of the angular displacement of a gears is approximately equal towards the percentage of the number of teeth. What is the connection between the input torque, T1, as well as the output torque, T2? When we assume the gears are lossless, that really is, they don't really absorb or energy stored, the energy entering Gear 1 equals the energy out of Gear 2.11 Ever since translational energy of force times displacement has now become the rotational energy of torque time's angular displacement,

Which implies the analogous arrangement at the input and without gears depicted. Thus, the load may be thought of as having been mirrored from the input onto the output. Generalizing the findings, we may formulate the following statement: Rotational mechanical impedances may be reflected across gear trains by increasing the mechanical susceptibility by the ratio,

$$\left(\frac{\text{Number of teeth of gear on } \textit{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textit{source} \text{ shaft}}\right)^2$$

Where the impedance to be reflected is connected to the source shaft and is being reflected towards the destination shaft. The following example highlights the application of the notion of reflected impedances as we determine the transfer function of a rotating mechanical system with gears.

Electromechanical System Transfer Functions

In the previous part we spoke about rotating systems using gears, which finished our examination of purely mechanical systems. Now, we turn to systems that really are hybrids of electrical and mechanical components, the electromechanical systems. Have seen one use of an electromechanical system in Volume 1, the antenna azimuth position monitoring system. Other uses for systems having electromechanical components include robot controls, sun and star trackers, including computer tape but also disk-drive position controllers. An example of the control system that employs electromechanical components. A motor is an electromechanical component that produces a displacement output for a voltage input that seems to be, a mechanical output created by an electrical input. We shall develop the transfer function for one specific sort of electromechanical system, namely armature-controlled dc servomotor (Mablekos, 1980).

$$v_b(t) = K_b \frac{d\theta_m(t)}{dt}$$

We call v(t) the back electromotive force (back emf); K is a constant of proportionality called the back emf constant; and dem(t)/dt = w(t) is the angular velocity of the motor. Taking the Laplace transform, we get

Vb(s) = Ks0m(s)

The relationship between the armature current, i(t), the applied armature voltage, ea(t), and the back emf, v(t), is found by writing a loop equation around the Laplace transformed armature circuit:

Rala(s) + Lasla(s) + Vb(s) = Ea($)

The torque developed by the motor is proportional to the armature current; thus,

Tm(s) = Kila(s)

Where T is the torque developed by the motor, and K, is a constant of proportionality, m called the motor torque constant, which depends on the motor and magnetic field characteristics. In a consistent set of units, the value of K, is equal to the value of K. Rearranging yields.

Ia(s) === Tm(s)

K□

To find the transfer function of the motor, it first substitute, yielding.

(Ra + Las)Tm(s) K□ ·+Kbs0m(s) = Ea(s)

We need to compute Tm…s† in terms of θm s if we would like to separate the variables into input and output and get the transfer function, θm s =Ea s . Figure 2.36 depicts a typical comparable mechanical stress on a motor. Jm is the analogous inertia there at armature and comprises the both armature inertia and, as they shall see later, total load inertia reflected towards the armature. Dm is indeed the approximate solution damping just at armature and comprises both of the armature viscous damping also, as we shall see later, the load viscous dampening reflected towards the armature.

Electric Circuit Analogs In this part, we highlight the commonality of systems from the many disciplines by proving that the mechanical systems with which we worked may be represented by analogous electric circuits. They have pointed out the resemblance between the equations derived from Kirchhoff's rules for electrical systems as well as the equations of motion of mechanical systems. We now illustrate this similarity even more strongly by creating electric circuit counterparts for mechanical systems.

The variables of the electric circuits operate identically like the comparable variables of the mechanical systems. In fact, transforming mechanical systems into electrical networks before developing the descriptive equations is a problem-solving method that you might wish to investigate. An electric circuit that is comparable to a system from some other discipline is termed an electric circuit analog. Analogs may be generated by comparing the descriptive equations, such as the equations of motion equations of such a mechanical system, either with the electrical mesh and nodal equations.

Whenever compared using mesh equations, the resultant electrical circuit is termed a series analog. Whenever compared using nodal equations, the resultant electrical circuit is termed a parallel analog.

Series Analog

Considering the translational mechanical system represented, whose motion's equation of motion is (Figure 3.2),

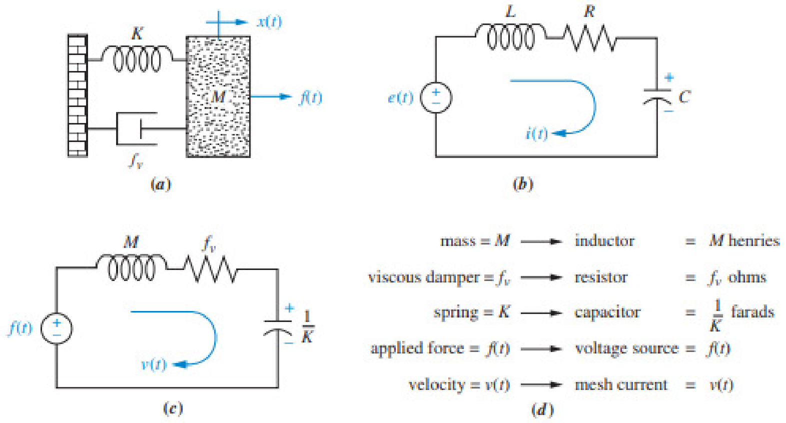


Figure 3.2 series analog

Operating on to convert displacement to velocity by dividing and multiplying the left-hand side by s, yielding.

$$\frac{Ms^2 + f_v s + K}{s} sX(s) = \left(Ms + f_v + \frac{K}{s}\right)V(s) = F(s)$$

Analog Parallel

A system may also be transformed into an analogous parallel version. Take into account the translational mechanical system represented, who's the motion equation. For the straightforward parallel RLC network seen, Kirchhoff's nodal equation is,

$$\left(Cs + \frac{1}{R} + \frac{1}{Ls}\right)E(s) = I(s)$$

We determine the total admittances and sketch the circuit illustrated in by comparing. It provides a summary of the conversions (d). The parts of a motion that include more than one degree of freedom are represented as parallel electrical elements linked to a node. Between two related nodes, the components of nearby movements are shown as parallel electrical elements. We provide an example to illustrate Figure 3.3.

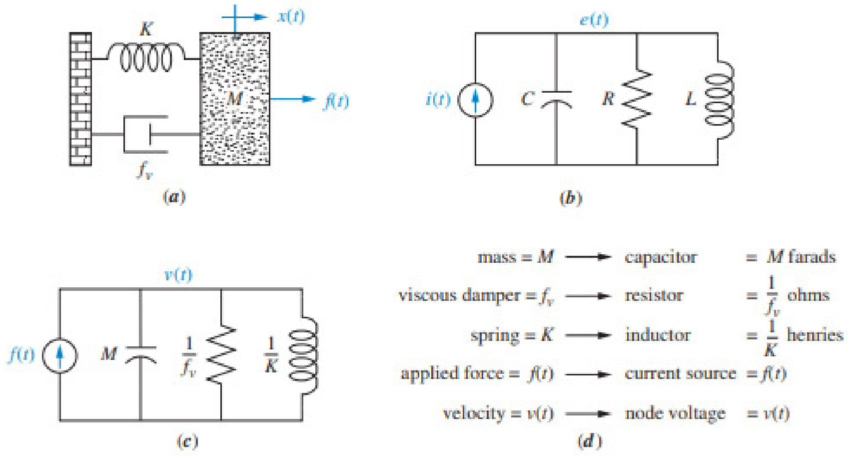


Figure 3.3 Parallel analog

Nonlinearities

The models created so far are based on systems that roughly correspond to those represented by linear, time-invariant differential equations. The creation of these models included an implicit linearity assumption. We officially define the words linear and nonlinear in this section and demonstrate how to differentiate between the two. We demonstrate how to represent a nonlinear system as a linear system. So that we may use the previously discussed modeling strategies. Two characteristics of a linear system are superposition and homogeneity. Due to the feature of superposition, a system's output reaction to the whole of its inputs is equal to the total of its individual input responses. In other words, if an input of r1...t produces an output of c1...t and an input of r2(t) produces an output of c2...t, then an input of r1...t combined with r2...t produces c1...t and c2...t, respectively. The reaction of the system to multiplying the input by a scalar is described by the homogeneity property. The property of homogeneity in a linear system is specifically shown if, given an input of r1...t that gives an output of c1...t, an input of Ar1...t yields an output of Ac1...t; that is, multiplying an input by a scalar results in a response that is also multiplied by a scalar. Linearity is shown as such. The output of the linear system is always (Figure 3.4).

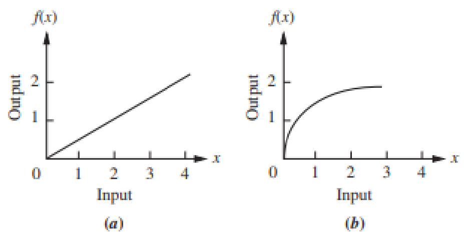


Figure 3.4 output of the linear system

Regardless of a value of x, f...x 0:5x, which is equal to half the input. Every one of the two linear system characteristics so holds true. As an example, an input of 1 results in an output of 1 and 2, whereas an input of 2 results in an output of 1. Superposition should result inside an output that is the total of the individual outputs, or 1.5, from an input that represents the sum of the original inputs, or 3. A 3 input results in a 1.5 output. Consider an input of 2 that also results in an output of 1, to test the homogeneity property. This input should result in an output that is twice as much, or 2. An input of 4 does in fact result in an output of 2. The connection shown couldn't possibly fit the criteria for linearity, as the reader may confirm (b).

Some examples of physical nonlinearities are shown. Although an electrical amplifier is linear within a certain range, excessive input voltages cause it to become nonlinear and display saturation. The term dead zone refers to a nonlinearity that occurs in a motor when frictional forces prevent it from responding at extremely low input voltages. Backlash is a nonlinearity that occurs in loosely fitting gears when the input moves across a narrow range even without output reacting. The reader should be sure that the curves in do not, across their full range, fulfill the requirements of linearity. A phase detector, which is utilized in a phase-locked loop inside an FM radio reception and whose output response equals the sine of the input, is another example of such a nonlinear subsystem. A nonlinear system may often be approximated linearly by a designer. As long even as outcomes provide a decent approximation to reality, linear approximations were utilized to streamline system development and analysis. For instance, if the origin is moved to a position on the nonlinear curve where the range between input values is minimal, a linear connection may be constructed at that location. Physical objects that conduct linear amplification with minute excursions around a point include electronic amplifiers.

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CHAPTER 4

LINEARIZATION

Mr. Tony Aby Varkey M, Assistant Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- tonyaby.varkey@presidencyuniversity.in

The mechanical and electrical systems discussed thus far were predicated on linearity. To obtain the transfer function, we must first linearize the system if it has any nonlinear components. Having defined and analyzed nonlinearities in the previous part; in this section, they demonstrate how to approximate nonlinear systems in linear form in order to derive transfer functions. Recognizing the nonlinear component and creating the nonlinear differential equation is the first step. For small-signal inputs concerning the steady-state solution where the small-signal input gets equal to zero, researchers linearize a nonlinear differential equation. Equilibrium is the name of this steady-state solution, and it is chosen as the second stage in the linearization procedure. For instance, a pendulum is in balance when it is in rest. Although the angular displacement is given by a nonlinear differential equation, it is possible to express tiny deviations from this equilibrium point using a linear differential equation. We then linearize the nonlinear differential equation and, assuming zero starting conditions, we obtain the Laplace transform of the linearized differential equation. Finally, we create the transfer function by separating the input and output variables. Let's look at how to linearize a function first, and then we'll use that knowledge to linearize a differential equation[1]–[6].

Small changes in the input may be attributed to changes in the output around the point by means of the slope of the curve at the point A if we assume a nonlinear system operating around point A, fx0; fx. Consequently, if the contour of the curve at point A equals ma, then minor deviations of the input (x) from point A will result in slight changes in the output (fox), which are connected by the slope at point A. Thus,

$$[f(x) - f(x_0)] \approx m_a(x - x_0)$$

from which

$$\delta f(x) \approx m_a \delta x$$

and

$$f(x) \approx f(x_0) + m_a(x - x_0) \approx f(x_0) + m_a \delta x$$

This connection is shown visually. At point A, a new pair of axes, x and f x, are produced, and f(x) is roughly equal to f x0, the ordinate of the new origin, plus minor detours, ma x. Let's examine a case in point.

Several common techniques

The majority of effective methods for numerical linear algebra rely on the use of orthogonal transformations. The SVD for rank determinations and generalized inverses, the Schur decomposition with eigenvalue and generalized eigenvalue issues, and the QR decomposition for least-squares problems are common instances of this. The majority of dependable linear algebra algorithms for control theory also use orthogonal transformations. This is largely because control issues have been directly addressed by existing linear algebra decompositions. Examples of this include using the Schur technique to solve continuous- and discrete-time algebraic Riccati equations, solve Lyapunov equations, and conduct pole placement. Additionally, new orthogonal decompositions have been published that strongly depend on the same ideas but were created especially for control-related issues. A system with A, B, and C is transformed into a new state-space representation with UHAU, UHB, and CU as a consequence of orthogonal state-space transformations, where U decomposes the matrices A, B, and C in some way. These unique shapes, often known as "condensed forms. Those orthogonal state-space transformations are used for two primary reasons: The numerical sensitivity of the control problem under consideration is not affected by these transformations even though sensitivity has been measured by norms or angles of specific spaces, and these are unaffected by orthogonal transformations. Orthogonal transformations have a minimum condition number, which is crucial in proving bounded error propagation but also establishing numerical stability of an algorithm that employs such transformations. This topic is covered in further length and later parts, wherein some of these condensed versions are employed for specific purposes.

Poles, zeros, and transfer functions

It discusses significant linear system structural characteristics and the numerical methods available to ascertain them. A state-space model C(I A) 1 B + D or a polynomial representation V()T1()U() + W() may both provide the transfer function R() of a linear system. Both the discrete-time situation (where stands for the shift operator z) and the continuous-time example (where stands for the differentiation operator D) are consistent with the conclusions in this subsection. Utilizing Polynomials the poles, transmission zeros, decoupling zeros, and other structural characteristics of the transfer function R() are of interest. All of this may be discovered using a rootfinder and a greatest common divisor (GCD) extraction algorithm in the scalar case, where T(, U(, V(, and W() are scalar polynomials. The issue gets far more complicated in the matrix situation, and the fundamental GCD extraction technique, the Euclidean algorithm, becomes unstable. The polynomial technique is less appealing than the state-space approach since additional structural components (such as null spaces, etc.) enter the picture. State-Space Methodology The poles and zeros of R(), decoupling zeros, controllable and unobservable subspaces, supremal (A, B)-invariant and controllable subspaces, factorizability of R(), left and right null spaces of R(), etc. are the structural features of importance. These ideas are crucial to many design issues and have attracted a lot of attention in recent years; for examples. All of the ideas listed above may be seen as generalized eigenstructure issues, and it is shown that they can be calculated using the Kronecker canonical form of the pencils.

$$[\lambda I - A] \qquad [\lambda I - A \mid B]$$

$$\left[\begin{array}{c} \lambda I - A \\ \hline -C \end{array}\right] \qquad \left[\begin{array}{c|c} \lambda I - A & B \\ \hline -C & D \end{array}\right]$$

Or from pencils that are descended from these. It is also possible to compute the Kronecker structure of any pencil using backward-compatible software. The fact that several of the structural qualities stated above may be ill-posed in situations where the concept of limited condition must be developed is a lingering issue in this situation. Reformulating the issue as an optimization or approximation problem, for which quantitative measurements are created, and letting the user make the ultimate decision represent an entirely new strategy. Results in this vein are achieved for controllability, observability, and controllability subspaces that are (nearly) (A, B)-invariant.

Controllability and other Abilities

The study of linear control and system theory is fundamentally based on the many "abilities" including controllability, observability, reachability, reconstructibility, stabilizability, and detectability. These ideas may also be understood in terms of the prior section's discussion of decoupling zeros, controllable and unobservable subspaces, controllability subspaces, etc. Our comments here are focused on the idea of controllability, although they are not restricted to it. There have been several algebraic and dynamic descriptions of controllability provided; see for an example. But when applied to finite arithmetic, each of these has issues. For a summary of this subject and several instances. The close connection between the controllability issue and the invariant subspace problem contributes to the difficulty of solving controllability numerically. The smallest A-invariant subspace (subspace covered by eigenvectors or main vectors) spanning the range of B is the controlled subspace related to Equation 1.1. It implies that the controlled subspace may be just as perturbation sensitive as the A-invariant subspaces. The calculation of the so-called controllability indices should be noted similarly. The slightly perturbed matrix A contains n eigenvectors associated with the n different eigenvalues, while matrix A only has one eigenvector (associated with 0). The pole placement issue, which is covered in a subsequent section, has been the subject of attempts to devise numerically stable methods. It is sufficient to establish here that controllability and the issue of pole placement via state feedback are closely connected. The reduction of A to a Hessenberg form forms the basis of work on creating numerically stable algorithms for pole placement; for examples. The controller Hessenberg form, where the input vector B is a multiple of (1, 0, 0) T and the state matrix A is upper Hessenberg, is a suitable strategy in the single-input scenario. If, and only if, all (n 1) sub-diagonal elements of A are nonzero, the pair (A, B) is then controllable. When a sub-diagonal element is 0, the system is uncontrolled, and it is simple to build a foundation for the uncontrollable subspace. From this "canonical form," the transfer function gain or first nonzero Markov parameter may likewise be simply created. In reality, the numerically more fragile special case of a companion or rational canonical or Luenberger canonical form is being replaced by the numerically more robust system Hessenberg form, which is becoming more and more important in system theory[7]–[11].

A topological concept like "near uncontrollability" is a more significant feature of controllability. However, there are additional numerical challenges in this case; for further information. This is related to the intriguing "balancing" system-theoretic idea that Moore's discusses. It discusses the calculation of "balancing transformations." There are at least two different concepts of near uncontrollability that apply to both energy and parametric systems. A controllable pair (A, B) is said to be near uncontrollable in the parametric sense if the parameters of (A, B) need to be altered just little for (A, B) to lose controllability. A controlled pair is almost uncontrollable in the energy sense if there is a lot of control energy. The pair

$$A = \begin{bmatrix} 0 & 1 & & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ & & & & \\ \vdots & & & \ddots & 1 \\ 0 & \cdots & & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

In terms of energy, it is very nearly uncontrolled, albeit not to the same extent as in terms of parameters. Of course, the coordinate dependence of both measurements exists, and "balancing" is an effort to correct for this bias. A and B are a pair that are in "controllable canonical form." As n becomes larger, it is now understood that matrices in this form, more especially the A matrix in rational canonical form, nearly invariably behave poorly numerically and become "close to" uncontrolled (unstable, etc.).

Pole assignment and observer design, an inverse eigenvalue issue is the design of state or output feedback for just a linear system to produce a closed-loop system with a specified set of poles. The following is the state feedback pole assignment problem: One searches for a matrix F such that the eigenvalues of the matrix AF = A + BF reside at certain points or in specific areas, given a pair (A, B). Many strategies have been explored to address this issue. However, the focus is on mathematically sound approaches and taking into account the problem's numerical sensitivity, as can be seen, for instance, in the publications. In observer design and deadbeat control with discrete-time systems (where A + BF is needed to be nilpotent), special instances of the pole assignment issue appear. The numerically accurate techniques for assigning poles are based on reducing A to an RSF, a Hessenberg or block Hessenberg (staircase). In contrast to the controlled or Luenberger canonical form, whose calculation is known to be numerically unreliable, the latter could be viewed as a statistically robust alternative. The extra flexibility provided by the state-feedback matrix for multi-input systems may be employed for eigenvector assignment and sensitivity reduction for the closed-loop poles. Instead of computing the resultant matrix AF directly in this case, an iterative method is used to obtain the matrices X and of the decomposition AF = XX1. The iteration seeks to optimize the orthogonality of the eigenvectors xi or reduce the sensitivity of the inserted eigenvalues i. It is more challenging to assign poles using output feedback, both conceptually and computationally. As a result, a limited number of numerically trustworthy algorithms are accessible. Other studies on pole assignment have focused on descriptor or generalized state-space systems. Finding matrices T, AK, and K such that TAK AT = KC, wherein the spectrum of AK is determined, is the observer design issue for a given state-space system (A, B, C). Equation 1.48 becomes AK = A + KC, resulting in a transposed pole placement issue, when one commonly sets T = I. This is because Equation 1.48 is an underdetermined (and nonlinear) problem inside the unknown parameters of T, AK, and K. The aforementioned pole positioning methods in this situation are inherently applicable. When assuming AK in Schur form, one may still solve Equation 1.48 through a recurrence connection even if T in reduced order design is non-square and therefore cannot be equated to the identity matrix.

Strong Control

The theory and practices of robust control have significantly advanced over the last ten years; for examples, see and its references. Though the field of robust control is still developing, attention to its numerical elements has only just started. Therefore, it is too soon to review trustworthy numerical methods in the field. In this part, we analyze the so-called H method, a recent discovery in robust control that has garnered a lot of interest in order to provide an idea of the flavor of the numerical and computational challenges involved. H and the associated structured single value approach have given engineers a strong basis for creating reliable linear system controllers. The controllers are reliable because they provide the intended system performance despite a high level of system uncertainty. In this section, we refer to the collection of appropriate real rational matrices of dimension n m as R(s) n m. A stable matrix G(s) R(s) nm's H norm is defined as,

$$\|G(s)\|_{\infty} := \sup_{\omega \in \mathbb{R}} \sigma_{\max}[G(j\omega)],$$

Where max refers to a (complex) matrix's biggest singular value. There are several iterative ways to compute this norm. In one method, a connection is made between the imaginary eigenvalues of the a Hamiltonian matrix produced from a state-space realization of G as well as the singular values of G(jw) (s). The H norm of G is then calculated using this result using an effective bisection procedure (s). Imagine a linear, time-invariant system defined by the statespace equations to explain the fundamental H method,

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t),$$
$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t),$$
$$y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t),$$

The early approaches for calculating opt either employed an iterative search using spectral factorization or addressing the ensuing Nehari issue or calculated the spectral norm of the related Hankel plus Toeplitz operator. The initial formulation of this question was in a input-output setup. The issue is expressed in terms of two algebraic Riccati equations which rely on a gain parameter in a state-space formulation for calculating opt that seems promising from the perspective of numerical computation. Then, with certain assumptions, it can be shown that three requirements must be met for a controller K(s) K to exist such that Tzw , specifically that stabilizing solutions exist again for two Riccati equations as well as the spectral radius of the product of the solutions is constrained by 2. The relevant controller K(s) may be found from the solutions of the Riccati equations if these requirements are met for a certain value of. The maximum over all suboptimal values of such that the three requirements are met is the optimum gain, opt. The method described above instantly offers a computing opt algorithm of the bisection kind. However, in the vicinity of the ideal value, such an algorithm may be exceedingly sluggish. Using a gradient technique, speedups near the solution may be obtained, as shown in. An approach that combines a gradient technique with bisection is derived using the behavior of the Riccati solution as a function of. As the ideal value of approaches, it has been noted in that the Riccati equation may become ill-conditioned.

Transfer function

This problem simplifies to straightforward algebraic equations that are rather straightforward to solve when we use Laplace transforms to translate it to the s-domain. A transfer function is a term used to describe this altered representation of the system in the s-domain. Let's now examine a transfer function's definition in greater detail. The transfer function for a linear time invariant (LTI) system, assuming that all starting conditions are zero, is the ratio of the output's Laplace transform to its input's Laplace transform. Mathematical model, the transfer function G(s) is defined as follows if C(s) is the Laplace transform of an output function and R(s) is the Laplace transform of both the input function: Let's look at this little illustration in Figure 4.1.

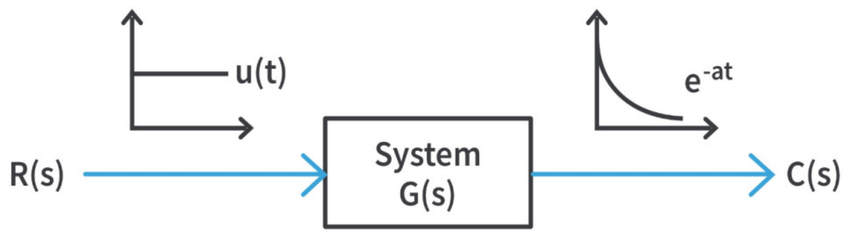

Figure 4.1: Illustrates the graphical presentation of transfer function.

The output is produced as indicated above after a unit step signal is provided as the input.

Let's use the Laplace transform and refer to this table.

The system's input is given as follows:

$$r(t) = u(t) \qquad \text{(a unit step signal)}$$

Which you get after using the Laplace transform.

$$R(s) = \frac{1}{s}$$

The output, on the other hand, is,

$$c(t) = e^{-at}$$

Taking the Laplace transform of this, we get,

$$C(s) = \frac{1}{s+a}$$

With this input and output relation, we find the transfer function is simply:

$$G(s) = \frac{C(s)}{R(s)} = \frac{\frac{1}{s+a}}{\frac{1}{s}} = \frac{s}{s+a}$$

Let's think about a different situation right now. What happens if R(s), the Laplace or s-domain constant, equals 1 Then G(s) equals C. (s). When the Laplace transform of something like the input is 1, the transfer function simply represents the Laplace transform of a output. This prompts the question, "Which input has a Laplace transform of 1" It is the impulse function or impulse signal.

A tall and thin signal known as a "impulse signal" (t) is produced as indicated (Figure 4.2).

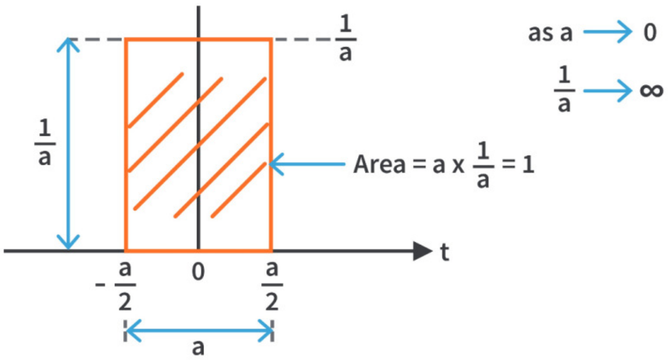

Figure 4.2 impulse signal

1/and becomes endlessly tall as an approaches absolute zero. This indicates that although if the rectangle's width and height are effectively zero and infinite, respectively, the rectangle's area is still one (Figure 4.3).

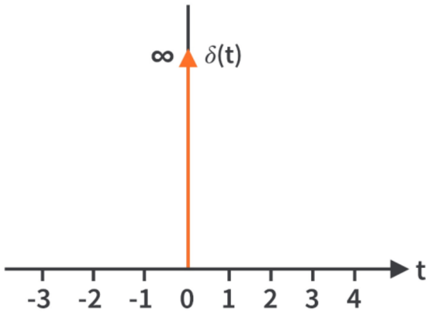

Figure 4.3 time vs impulse signal

Therefore, for any values of time other than t = 0, the impulse signal or impulse function has a value of 0.

Since the area is still 1, we may state,

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

The output that results from applying this impulse signal even as system's input is known as the impulse response. In real life, an impulse signal is similar to a brief, transient disturbance of the system, and an impulse response is how the system would normally respond to this disturbance in Figure 4.4. For the sake of clarity, let's take an example.

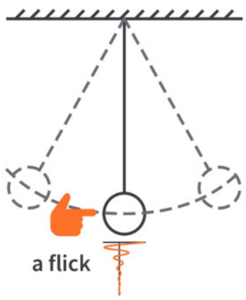

Figure 4.4: Illustrates the calculating the Impulse Reponses.

An impulse is created when your flick the pendulum using your fingertips. And the pendulum's impulse reaction determines how it responds. As we previously established, if the input is assumed to be impulse (t), then its Laplace transform equal,

$$R(s) = L\{\delta(t)\} = 1$$

And as per the definition of transfer function,

$$G(s) = \frac{C(s)}{R(s)} = \frac{C(s)}{1} = C(s)$$

In the case of an impulse input, the transfer function is thus the same as the output. With these two instances, students can see how the transfer function connects the input and output. In light of this, the transfer function is just the Laplace transform of a result produced after exciting an LTI system with an impulse signal.

Identifying the Transfer Function: Steps to Take

With the aid of the simple RLC circuit that we modeled in the last lesson, we will explore the procedures to be followed in order to establish the transfer function of a system in Figure 4.5.

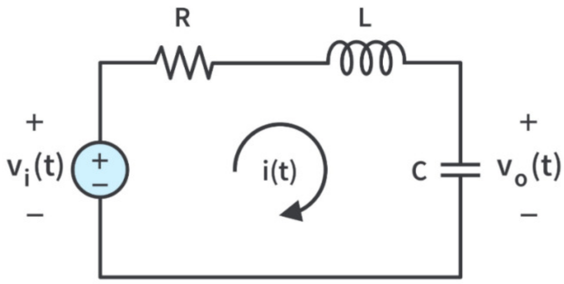

Figure 4.5: Illustrates the circuit diagram of identifying the transfer function.

Determine the supplied systems mathematical model equations in step 1.

The loop depicted above is subjected to Kirchhoff's voltage law,

$$v_i(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

$$v_o(t) = \frac{1}{C}\int i(t)dt$$

Step 2: Define the input and output variables for the system.

The input in this case is vi(t), and the output is vo(t).

Step 3: Using Laplace transforms and the assumption that the initial conditions are zero, convert the input and output equations into the s-domain. In this illustration, we'll suppose that the starting voltage across the capacitor and the initial current through into the inductor are both zero.

Consider the Laplace transform of an output and input equations that were obtained now. These are quite straightforward, so we can do a direct translation using the table.

$$V_i(s) = RI(s) + sLI(s) + \frac{1}{sC}I(s) = \left(R + sL + \frac{1}{sC}\right)I(s)$$

$$V_o(s) = \frac{1}{sC}I(s)$$

Step 4: Obtain the ratio of the Laplace transform of the output to the Laplace transform of the input.

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}I(s)}{\left(R + sL + \frac{1}{sC}\right)I(s)} = \frac{\frac{1}{sC}}{\left(R + sL + \frac{1}{sC}\right)} = \frac{1}{s^2LC + sRC + 1}$$

In conclusion, after identifying the equations in the time domain, we calculated their Laplace transform while supposing that the starting conditions are zero. The transfer function is then obtained by dividing the output by the input. To determine the transfer function of the majority of systems, one may follow this basic process.

Here are some crucial things to remember in relation to transfer functions:

The reaction of the system to different inputs and the nature of the system, which we will be studying in the next lessons, can both be studied using transfer functions.

The kind and size of the input have no bearing on transfer functions.

Transfer functions don't reveal anything about the system's make-up. It follows that distinct systems might have the same transfer function.

Transfer functions are often shown as in the diagram.

$$G(s) = \frac{C(s)}{R(s)} = \frac{x_0 s^m + x_1 s^{m-1} + \cdots\cdots + x_{m-1}s + x_m}{y_0 s^n + y_1 s^{n-1} + \cdots\cdots + y_{n-1}s + y_n}$$

$$G(s) = \frac{K(s-z_1)(s-z_2)(s-z_3)\cdots\cdots(s-z_m)}{(s-p_1)(s-p_2)(s-p_3)\cdots\cdots(s-p_n)}$$

Here,

n is the order of the system
K is the system gain (A proportional value)
$z_1, z_2, \ldots\ldots, z_m$ are the zeros of the system and
$p_1, p_2, \ldots\ldots, p_n$ are the poles of the system

The transfer function's poles are nothing more than the roots of its denominator polynomial. Similarly to this, the zeros are the transfer function's numerator polynomial's solutions. To put it another way, the poles and zeros are the values of s whereby the transfer function has become infinite and zero, respectively.

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CHAPTER 5

MODELING AND TRANSFER FUNCTION

Mr. Tony Aby Varkey M, Assistant Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- tonyaby.varkey@presidencyuniversity.in

Having modeled a DC motor at the conclusion of the prior lesson, from which we were able to get the model equation:

$$\frac{LJ}{K_T}\frac{d^3\theta}{dt^3} + \left(\frac{LB}{K_T} + \frac{RJ}{K_T}\right)\frac{d^2\theta}{dt^2} + \left(\frac{RB}{K_T} + K_b\right)\frac{d\theta}{dt} = v$$

Where L is the motor winding's inductance, R is its resistance, B is its damping coefficient, KT is its torque constant, KB is its back emf constant, v is its input voltage, and is the angle at which it rotates.

Let's assume all the constants are equal to one for the sake of simplicity.

Laplace transformation, we get

$$V(s) = s^3\theta(s) + 2s^2\theta(s) + 2s\theta(s) = (s^2 + 2s + 2)\ s\theta(s)$$

Let the speed of the motor be considered as the output, so we know:

$$\omega = \frac{d\theta}{dt}$$

Taking the Laplace transform of the output,

$$\omega(s) = s\theta(s)$$

Now, per the definition of transfer function,

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{s\theta(s)}{(s^2 + 2s + 2)\ s\theta(s)} = \frac{1}{s^2 + 2s + 2}$$

As a result, they have a transfer function for a DC motor. We will now use Scilab XCOS to model this transfer function.

Refer to the instruction shown below to get started with XCOS in Figure 5.1:

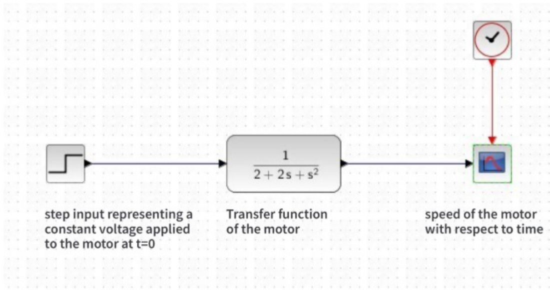


Figure 5.1: Illustrates the step wise modelling of transfer function.

And when the simulation is being conducted, we discover in Figure 5.2.

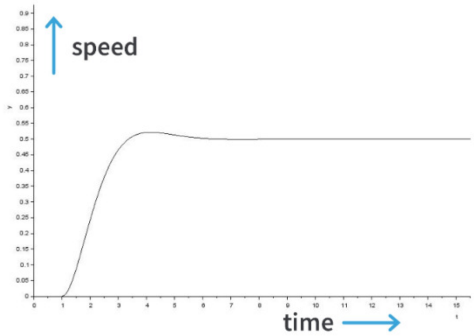


Figure 5.2: Graphical representation between the speeds versus time.

The only conclusion we need to draw from simulating the above model is that, as we apply a specific voltage to a motor, this same speed of the motor gradually increases from zero and stabilizes at a constant value. As a result, the transfer function can be utilized to analyze how the system behaves when subjected to different inputs. Let's not worry too much about the reaction right now since we will analyze the system's response in depth in the next lessons. In this lesson, we began by creating a transfer function. Next, using the Laplace transform table as just a guide, we took the voltage input and output of the RLC circuit and used it to calculate the transfer function for just a series RLC circuit. Then, we studied certain transfer function characteristics and discovered what poles and zeros were. After modeling a DC motor in the prior lesson, we finally retrieved its transfer function and examined its simulated response[1]–[6].

Representation of a Transfer Function

Block schematics

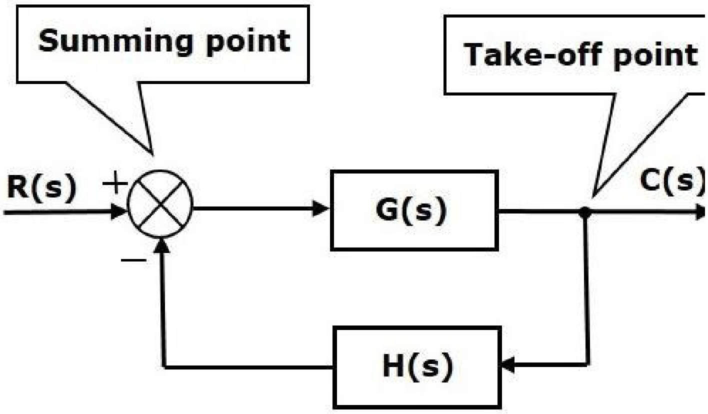
Block diagrams may be made up of a single block or several blocks. These are used to visually illustrate the control systems.

The take-off point, the summation point, and the block are the three fundamental components of a block diagram. To recognize these components, let's look at the block diagram of the closed loop control system as illustrated in the accompanying image in Figure 5.3

.

Figure 5.3: Illustrates the basic element of bloc diagram of takeoff point.

There are two blocks with transfer functions G(s) and H in the block diagram above (s). Additionally, it has a takeoff point and a summation point. Arrows show the direction of the signal flow. Let's now go through each of these components individually.



Block

A block represents a component's transfer function. One input and one output are present in the block.

A block with input X(s), output Y(s), as well as the transfer function G is shown in the accompanying image (s).

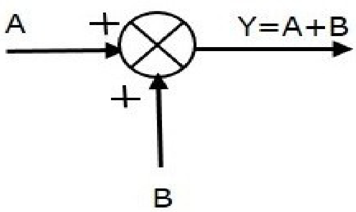
Transfer Function, $G(s) = \frac{Y(s)}{X(s)}$

$$\Rightarrow Y(s) = G(s)X(s)$$

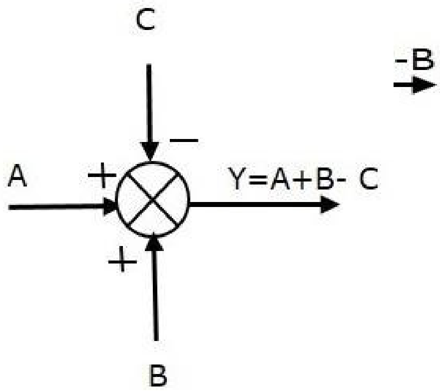
SummingPoint

A circle with an X within it serves as a visual representation of the summation point. It has one output and two or more inputs. It generates the inputs' algebraic sum. According on the polarity of the inputs, it also performs summation, subtraction, or a mix of summation and subtraction on the inputs. Let's examine each of these three procedures separately.The summing point having two inputs (A, B) and one output is shown in the following image (Y). In this case, both inputs A and B are positive. The output of the summing point is thus Y, which is equal to the total of A and B.The summing point having two inputs (A, B) and one output is shown in the following image (Y). In this case, the inputs A and B have the opposite signs, meaning that A is positive and B is negative. As a result, the output Y from the summing point is the difference between A and B, or.

Y=A+(-B)=A-B.



The summation point with three inputs (A, B, and C) and one output is shown in the following image (Y). The inputs A and B were positive in this case, whereas C is negatively skewed. The result of the summing point is output Y as,



Y=A+B+(−C)=A+B−C.

Take-offPoint

The take-off point is when more than one branch may be reached using the same input signal. That implies that we may apply the same input to one or more blocks, summing points, with both the aid of the take-off point. The take-off point is utilized in the following illustration to link the same input, R(s), to two more blocks in Figure 5.4.

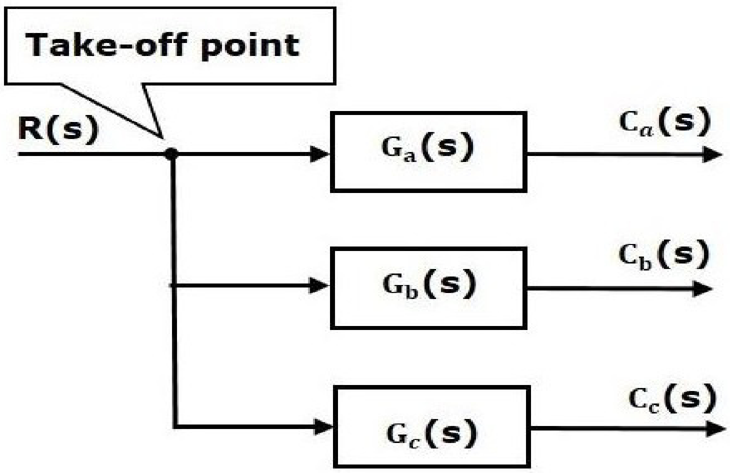


Figure 5.4: The take-off point is utilized in the following illustration to link the same input, R(s), to two more blocks.

The output C(s) are connected to the take-off point in the accompanying diagram as being one of the inputs to the summing point in Figure 5.5.

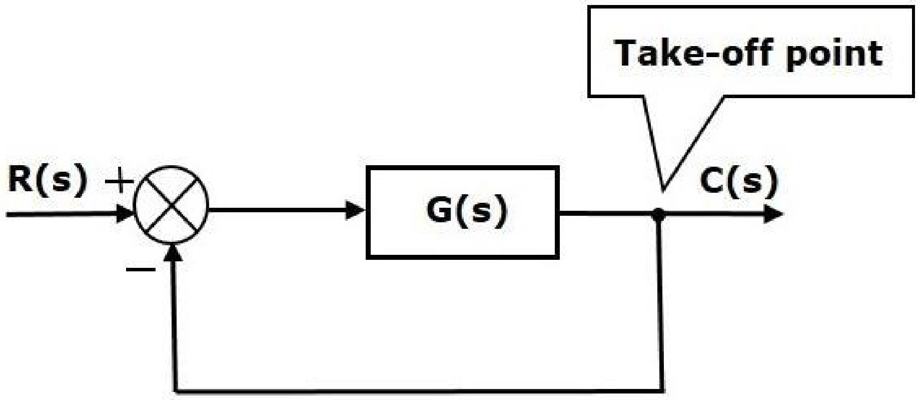


Figure 5.5: Illustrates the output C(s) are connected to the take-off point in the accompanying diagram as being one of the inputs to the summing point.

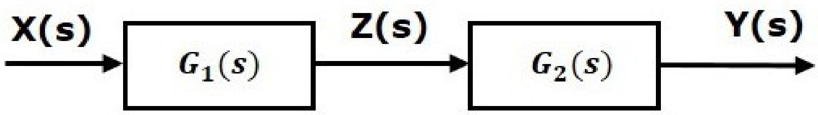
The algebra entailed with the fundamental components of the block diagram is known as "block diagram algebra." This algebra is concerned with the depiction of algebraic equations in pictures[7]–[11].

Basic Block Connections

The three most common sorts of relationships between two blocks are as follows.

Series Relationship

Cascade connection is another name for a series connection. Two blocks with transfer functions G1(s) G1(s) and G2(s) G2(s) are linked in series in the following diagram.



$$Y(s) = G_2(s)Z(s)$$

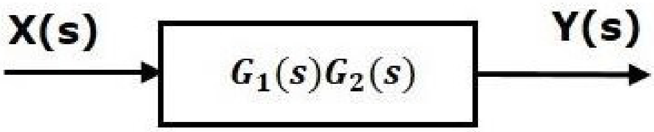
Where, $Z(s) = G_1(s)X(s)$

$$\Rightarrow Y(s) = G_2(s)[G_1(s)X(s)] = G_1(s)G_2(s)X(s)$$

$$\Rightarrow Y(s) = \{G_1(s)G_2(s)\}X(s)$$

Compare this equation with the standard form of the output equation, $Y(s) = G(s)X(s)$. Where, $G(s) = G_1(s)G_2(s)$.

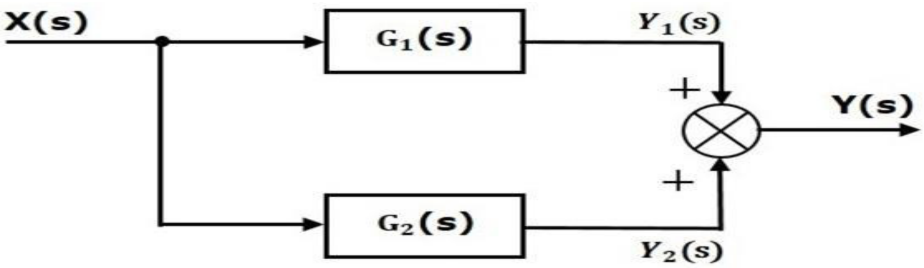
This indicates that we may use a single block to represent the series connection of two blocks. The transfer functions of the two blocks are added together to form the transfer function of the single block. Below is the corresponding block diagram.



Similar to this, a single block may be used to symbolize a series connection of 'n' blocks. The sum of the transfer functions of the 'n' blocks makes up the transfer function of the particular block.

Parallel Relationship

The input for all parallel-connected blocks will be the same. Two blocks with transfer functions G1(s) G1(s) and G2(s) G2(s) are linked in parallel in the following diagram. These two blocks' outputs are linked to the summing point.



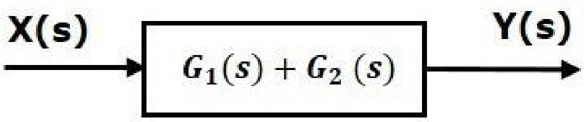
$$Y(s) = Y_1(s) + Y_2(s)$$

$$Y_1(s) = G_1(s)X(s) \text{ and } Y_2(s) = G_2(s)X(s)$$

$$\Rightarrow Y(s) = G_1(s)X(s) + G_2(s)X(s) = \{G_1(s) + G_2(s)\}X(s)$$

$$G(s) = G_1(s) + G_2(s).$$

This indicates that we may use a single block to represent the parallel connection of two blocks. The transfer functions of those two blocks are added together to form the transfer function of that kind of single block. Below is the corresponding block diagram.

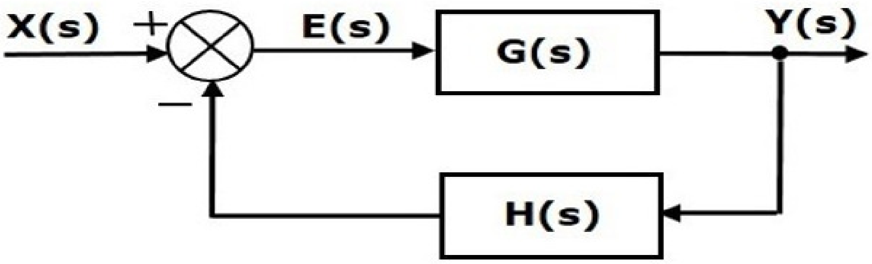


The parallel connection of 'n' blocks may also be represented by a single block. The algebraic sum of the transfer functions of each of those "n" blocks makes up the transfer function of this particular block

.

Feedback Relationship

There are two sorts of feedback: positive feedback and negative feedback, as we covered in earlier chapters. An example of a negative feedback control system is shown below. Here, a closed loop is formed by two blocks with the transfer functions G(s)G(s) and H(s)H(s).



$$Y(s) = \{X(s) - H(s)Y(s)\}G(s)\}$$

$$Y(s)\,\{1 + G(s)H(s)\} = X(s)G(s)\}$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Therefore, the negative feedback closed loop transfer function is:

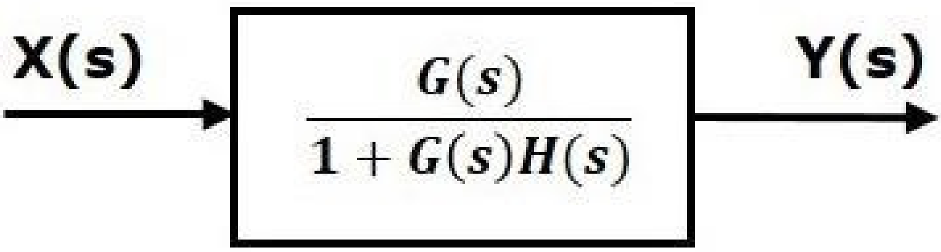
$$\frac{G(s)}{1+G(s)H(s)}$$

Therefore,thenegativefeedbackclosedlooptransferfunctionis:

$$\frac{G(s)}{1+G(s)H(s)}$$

This indicates that we may use a single block to represent the negative feedback relationship between two blocks. The closed loop transfer function of both the negative feedback serves as the transfer function of the single block. Below is the corresponding block diagram.

The positive feedback link between two blocks may also be represented by a single block. This particular block's transfer function is the closed-loop transfer function of both the positive feedback, that is.



$$\frac{G(s)}{1-G(s)H(s)}$$

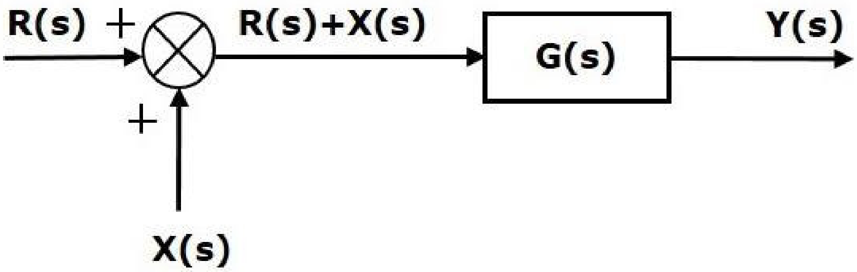
Block Diagram for Summing Points in Algebra

Moving summing point before the block and changing summing point after the block are the two options for shifting summing points with regard to blocks.

Let's examine the necessary preparations in each of the aforementioned two scenarios one at a time.

Summing Point before Block to After Block Shift

Take a look at the block diagram in the accompanying graphic. In this case, the block is preceded by the summing point.



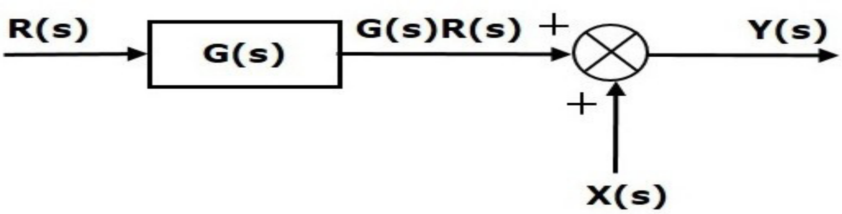
TheoutputofSummingpointis $\{R(s)+X(s)\}$.

Summing point has two inputs $R(s)$ and $X(s)$

So, the input to the block $G(s)$ is $\{R(s)+X(s)\}$ and the output of it is –

$$Y(s) = G(s)\{R(s)+X(s)\}$$

$$\Rightarrow Y(s) = G(s)R(s) + G(s)X(s) \qquad \text{(Equation 1)}$$



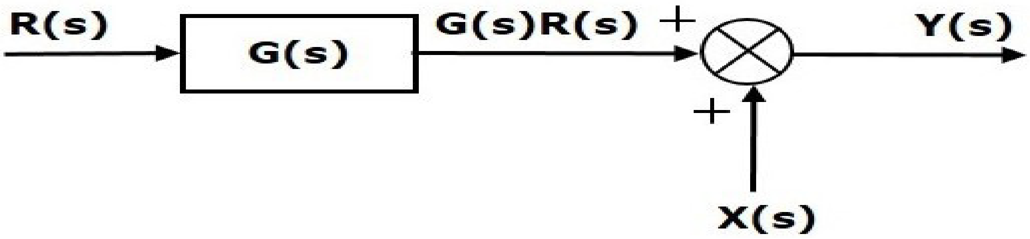
Output of the block $G(s)$ is $G(s)R(s)$.

The output of the summing point is

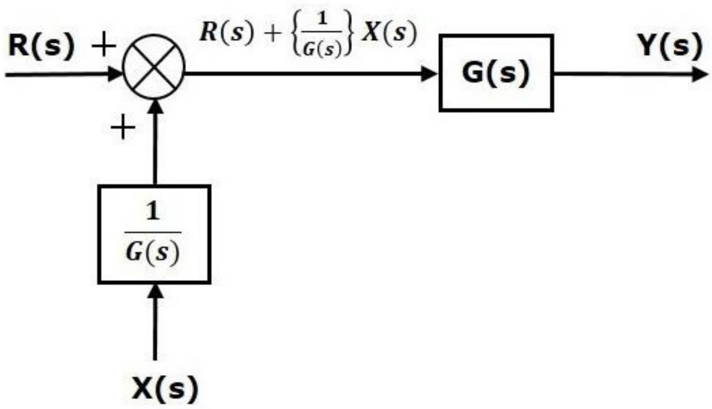
$$Y(s) = G(s)R(s) + X(s) \qquad \text{(Equation 2)}$$

Equations 1 and 2 should be compared.

The initial term in both equations is "G(s)R(s)" ("G(s)R(s)"). But the second term differs from the first. We need one more block G(s)G in order for the second term to be the same (s). Its input is X(s)X(s), and its output is used as the summation point's input rather than X(s)X. (s). The next figure displays this block diagram.



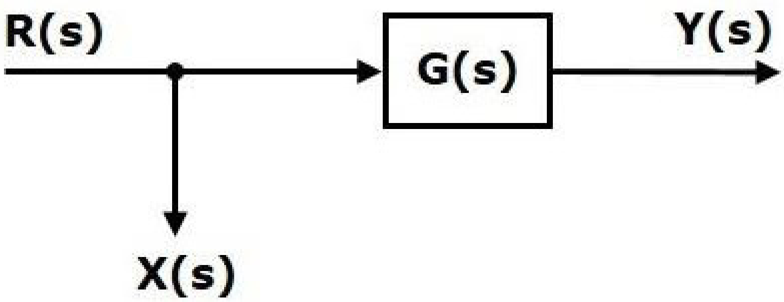
In both equations, the initial term, G(s) R(s), remains the same. But the second term differs from the first. Only need one more block 1/G to get the second term to be the same (s). It has the input X(s), and its output is used as the input to a summing point in place of X. (s). The next figure displays this block diagram.



Algebraic Block Diagrams for Takeoff Points

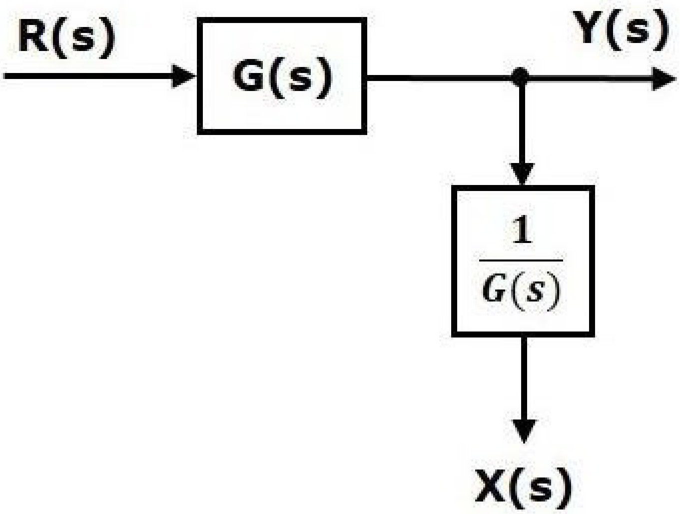
Two options exist for moving the take-off points in relation to blocks: moving the take-off point even before block and moving the take-off point after the block.

Let's examine the appropriate arrangements in each of the aforementioned two scenarios one at a time. Changing the takeoff point from being before a block to being after the block Take a look at the block diagram in the accompanying graphic. The take-off point is available in this instance before.

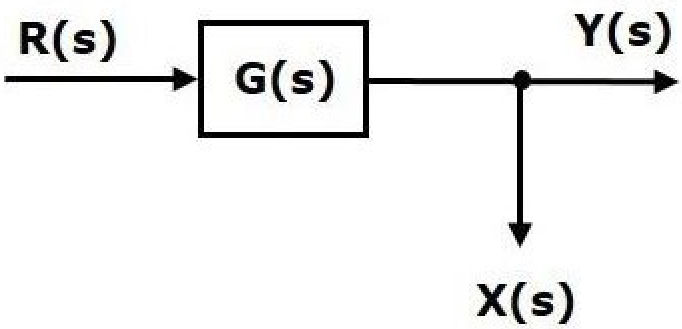


Here, $X(s) = R(s)$ and $Y(s) = G(s)R(s)$

The output Y(s) will be the same if you move the take-off point just after block. However, the value of X(s) differs. Consequently, we need one more block 1/G to obtain the same value of X(s) (s). Its input is Y(s), and its output is X. (s) the next figure displays this block diagram.

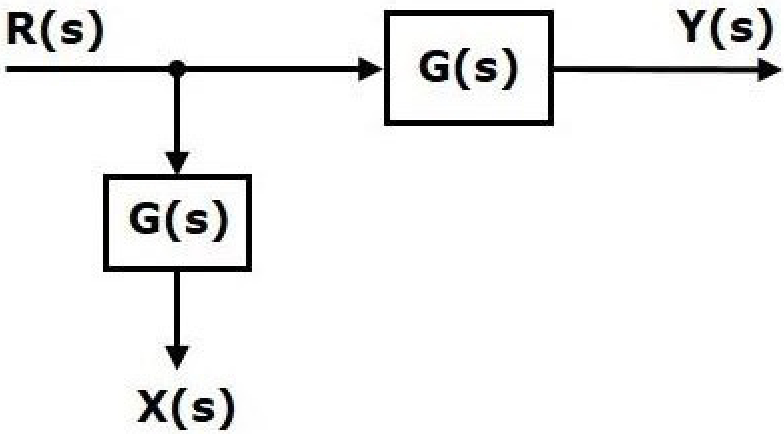


Changing the takeoff point from one that is after a blocks to one that is before the block Take a look at the block diagram in the accompanying graphic. In this case, the takeoff point is after the block.



Here, $X(s) = Y(s) = G(s)R(s)$

The output Y(s) will be the same if visitors move the take-off point before the block in. However, the value of X(s) differs. In addition to get the same value for X(s), we thus need block G. (s) Its input is R(s), and its result is X. (s). The next figure displays this block diagram.



The preceding chapter's techniques are useful for condensing (simplifying) the block diagrams.

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CHAPTER 6

DERIVATIVE, PROPORTIONAL, AND INTEGRAL CONTROL

Dr. Sreenivasappa B V, Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- sreenivasappabv@presidencyuniversity.in

In the last session, we learned the fundamentals of how a controller or a compensator operates. The "Proportional, Integral and Derivative Control" (PID Control), which is perhaps one of the most used control approaches, will be covered in this lesson.

Below is a generic block diagram of the a system using PID control in Figure 6.1,

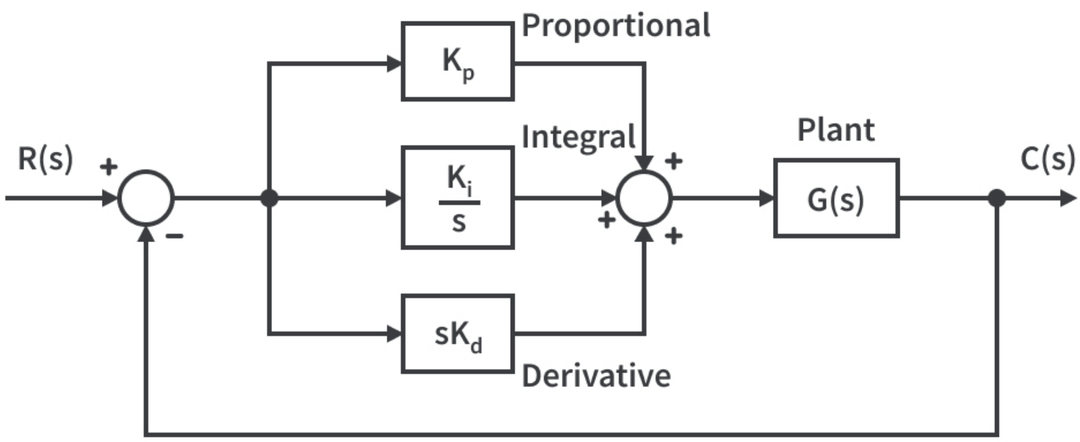


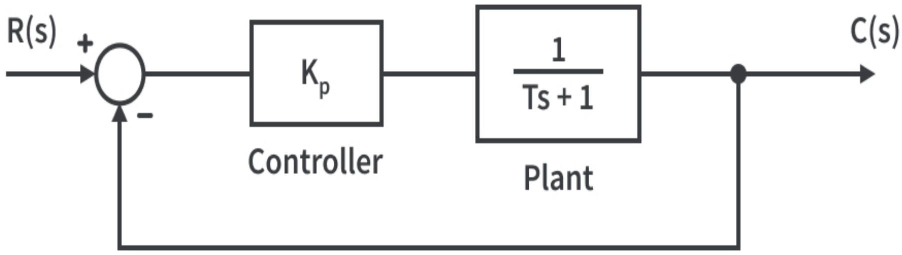
Figure 6.1: Illustrates theblock diagram of the system using PID control.

Starting with proportional control, we'll go through each kind individually.

Quantitative Control

The proportional control delivers the control action that is proportionate to the error present, acting on the current error. Take a broad illustration to help illustrate the point. Consider that you wish to go by foot from point A to point B. The error (difference between the intended location and the present position) increases as you walk, which causes you to move more quickly. But as you grow closer to point B, things start to slow down, and when you get there, simply stop moving. Ones speed in this situation is directly inversely proportional to the inaccuracy. This method of proportionally regulating your pace depending on the inaccuracy is known as control[1]–[6].

Consider the following illustration of a first-order plant with proportional control,



The open loop transfer function for the plant above is

$$G(s) = \frac{K_p}{T_s + 1}$$

and the closed loop transfer function is

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{K_p}{Ts + 1 + K_p} = \frac{\frac{K_p}{1+K_p}}{\frac{T}{1+K_p}s + 1} = \frac{K'}{T's + 1}$$

For a unit step input as reference, let's find the steady state error that the plant produces.

The error is given by E(s),

$$E(s) = \frac{1}{1 + G(s)}R(s) = \frac{Ts + 1}{Ts + 1 + K_p}\frac{1}{s}$$

The steady state error e_{ss} is given by

$$e_{ss} = sE(s) = \lim_{s \to 0} \frac{s(Ts + 1)}{Ts + 1 + K_p}\frac{1}{s} = \frac{1}{1 + K_p}$$

These equations show that the system's time constant has decreased from T to with the addition of proportional control,

$$\frac{T}{K_p + 1},$$

Consequently, the system now reacts considerably more quickly. Another finding is that when Kp increases, the steady state error decreases inFigure 7.2. However, as we observed in the lesson on the root locus plot, raising Kp in higher order systems causes the poles to shift to the right side of the s plane, rendering the system unstable. We will utilize Scilab's XCOS as primary simulation tool for this course. Because it contains a built-in PID block, it is easier for us.

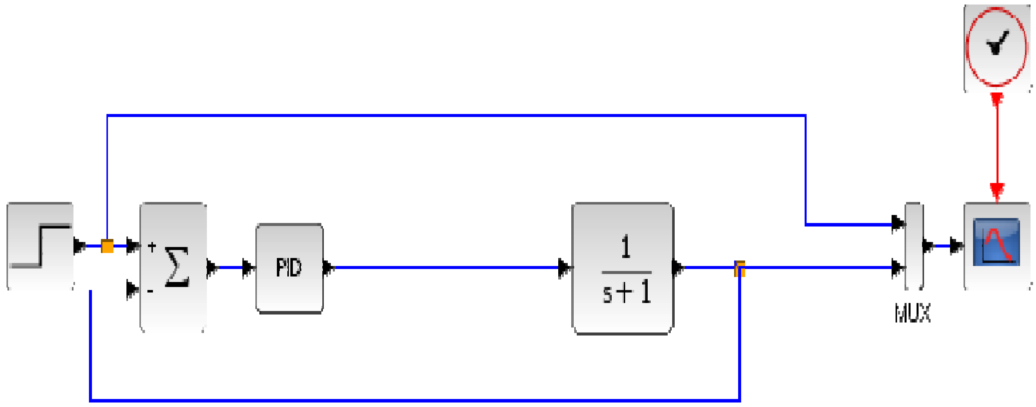


Figure 6.2: Illustrates the steady state error decreases.

This window appears when visitors click on the PID control block inFigure 6.2. The settings may be set here. We first simulate without such proportional control by setting Kp = 1 and maintaining 0 for the others. The outcome we get is.

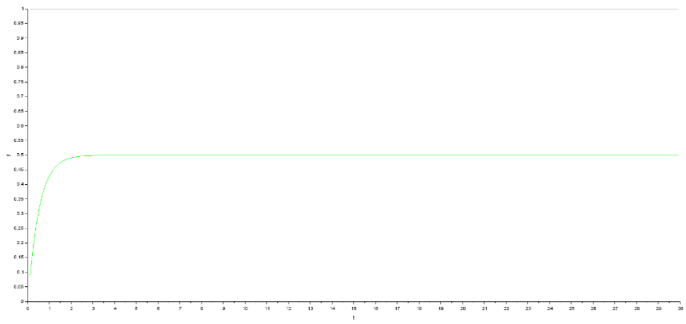


Figure 6.3: Illustrates the window appears when visitors click on the PID control block.

As can be seen, the steady state inaccuracy is considerable. (The black one is the reference input, while the green one represents the response in Figure 6.3.) Let's imitate proportional control; for this, we'll set Kp = 20, keeping the other values at 0. The outcome we get is (Figure 6.4),

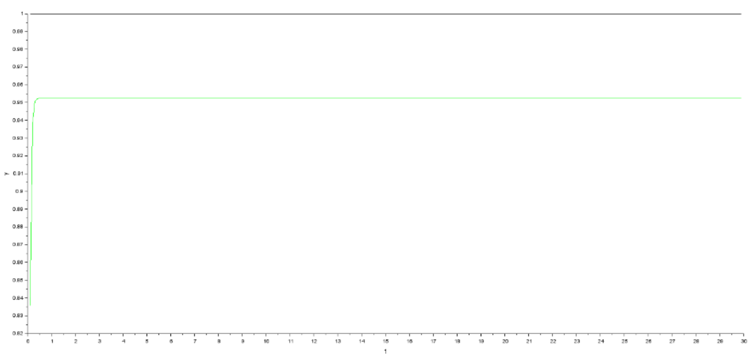


Figure 6.4: Illustrates the black one is the reference input, while the green one represents the response.

What we described before is supported by the system's improved response time and decreased steady state error.

Control Integral

Integral control does not address the immediate problem but rather the accumulated error inFigure 6.5. This implies that even if the mistake eventually decreases to zero, the control action doesn't really, and instead stays constant. A drone or a helicopter might serve as an illustration of this. Say you want to ascend to a height of X meters above the earth. Once you reach that height, you want to keep the propellers' speed constant and leave them running. The propellers' speed saturates and stabilizes as the inaccuracy decreases. You employ an integral in this kind of circumstance[7]–[10].

Consider the following example of a first order plant or system with integrated control,

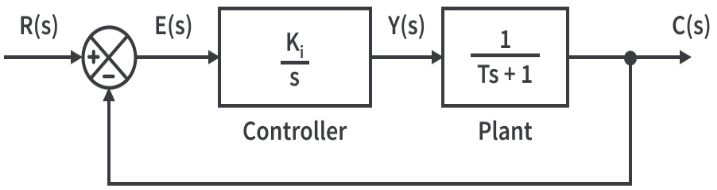


Figure 6.5: Illustrates the Integral control does not address the immediate problem but rather the accumulated error.

The output of the controller is the integral of the error signal.

$$Y(s) = \frac{K_i}{s} E(s)$$

Which is nothing but

$$y(t) = K_i \int_{-\infty}^{t} e(t)\,dt.$$

The open loop transfer function for the plant above is

$$G(s) = \frac{K_i}{s(Ts+1)}$$

and the closed loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K_i}{s(Ts+1)+K_i}$$

We'll calculate the steady state error for just a unit step input as a reference, just as we did before.

E provides the error (s),

$$E(s) = \frac{1}{1+G(s)} R(s) = \frac{s(Ts+1)}{s(Ts+1)+K_i} \frac{1}{s}$$

The steady state error e_{ss} is given by

$$e_{ss} = sE(s) = \lim_{s \to 0} \frac{s(Ts+1)}{s(Ts+1)+K_i} = 0$$

With the integral control, we are now able to completely make the steady state error 0, but this comes at a cost of increasing the order of system (originally the system in this example was of first order, butwith integral control, G(s) is second order, and we have seen in the tutorial on root locus that adding an open loop pole to a higher order system can make the system unstable in Figure 6.6. Hence, there is this risk. Most of the time, integral control is used along with proportional control, and it's called proportional – integral control. We shall come to the reason for this shortly.

To simulate this, we take the same block diagram, but here we set $K_i = 1$ and all others 0. Let's look at the response,

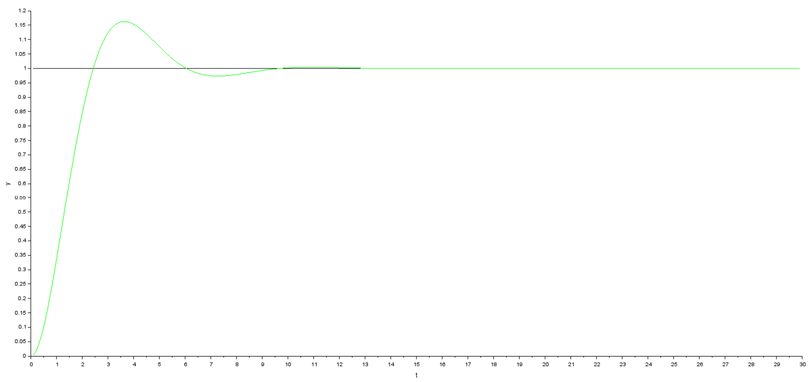


Figure 6.6: Illustrates an open loop pole to a higher order system can make the system unstable.

Since the system's order has now grown to two, the steady state error in this case is 0, and the first detection of a little oscillation confirms our argument.

Now let's look at a second order system,

$$G(s) = \frac{1}{s(s+10)}$$

A block schematic of the system using proportional control and an external disturbance is also shown below (s) inFigure 7.6.

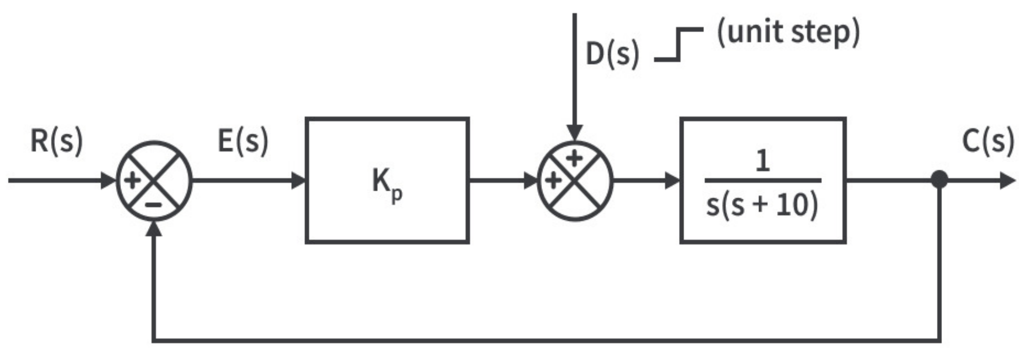


Figure 6.7: Illustrates a block schematic of the system using proportional control and an external disturbance.

Setting the input R(s) to 0, let's now calculate the transfer function connecting the disturbance D(s) towards the output C(s) inFigure 6.8.

$$\frac{C(s)}{D(s)} = \frac{\frac{1}{s(s+10)}}{1+\frac{K_p}{s(s+10)}} = \frac{1}{s^2+10s+K_p}$$

Now the error $E(s)$ is given by,

$$E(s) = R(s) - C(s) = 0 - C(s) = -C(s) = -\frac{1}{s^2+10s+K_p}D(s)$$

If we assume $D(s)$ to be a unit step function, then

$$E(s) = -\frac{1}{s^2+10s+K_p}\frac{1}{s}$$

The steady state error will be

$$e_{ss} = sE(s) = \lim_{s\to 0}\frac{-s}{s^2+10s+K_p}\frac{1}{s} = \frac{-1}{K_p}$$

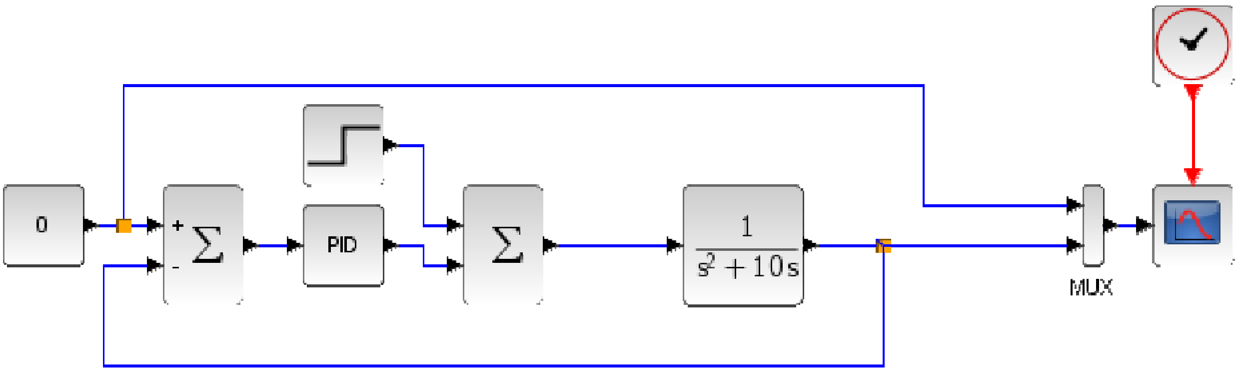


Figure 6.8: Illustrates the transfer function connecting the disturbance D(s) towards the output C(s).

Here, we will set Kp = 1000 and observe the outcome in order to significantly reduce the steady state error.

They first see high frequency oscillations despite the almost low steady state inaccuracy. As a result, by raising Kp, the steady state may be decreased, but as a tradeoff, the system begins to oscillate more (at a higher natural frequency), which might compromise stability inFigure 6.9. We will now combine proportional control plus integral control within the same system to examine how things change from of the previous situation.

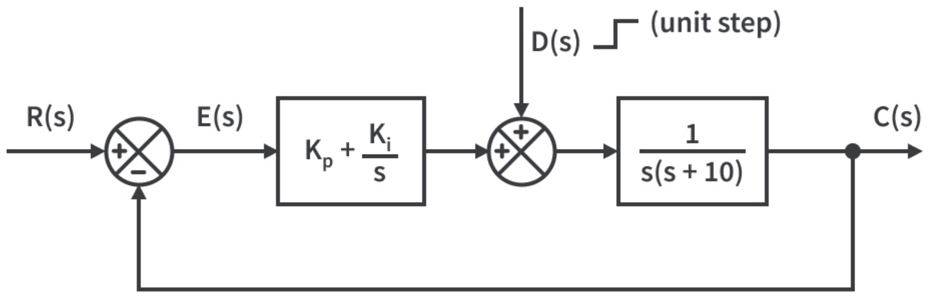


Figure 6.9: Illustrates the Combine proportional control plus integral control within the same system.

Let's now compute the transfer function relating disturbance $D(s)$ to the output $C(s)$ by setting input $R(s)$ to 0.

$$\frac{C(s)}{D(s)} = \frac{\frac{1}{s(s+10)}}{1 + \frac{K_p + \frac{K_i}{s}}{s(s+10)}} = \frac{s}{s^3 + 10s^2 + K_p s + K_i}$$

Now the error $E(s)$ is given by,

$$E(s) = R(s) - C(s) = 0 - C(s) = -C(s) = -\frac{s}{s^3 + 10s^2 + K_p s + K_i} D(s)$$

If we assume D(s) to be a unit step function, then

$$E(s) = -\frac{s}{s^3 + 10s^2 + K_p s + K_i} \frac{1}{s}$$

The steady state error will be

$$e_{ss} = sE(s) = \lim_{s \to 0} \frac{-s^2}{s^3 + 10s^2 + K_p s + K_i} \frac{1}{s} = 0$$

Here, they should choose Kp and Ki to ensure system stability. Utilizing the Routh array.

s^3	1	K_p
s^2	10	K_i
s	$(10K_p - K_i)$	
1	K_i	

As 10 * 10 > 20 in this case, let's select Kp = 10 and Ki = 20 for stability to get observations replicating this scenario.

As a result, humans can conclude that perhaps the proportional and integral control worked well to eliminate the steady state error without becoming very oscillatory. Proportional-integral (PI) control effectively rejects the disturbance (Figure 6.10).

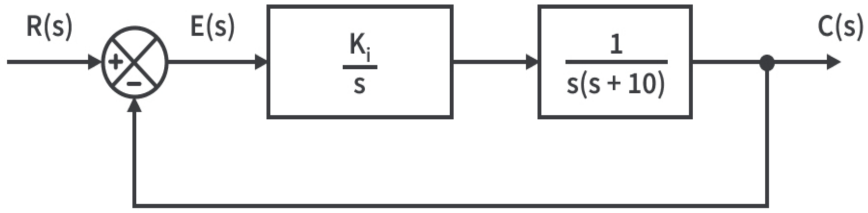


Figure 6.10: Illustrates the Proportional-integral (PI) control effectively rejects the disturbance.

The open loop transfer function again for aforementioned system changes to.

$$G(s) = \frac{K_i}{s^2(s+10)}$$

The genesis of this currently has two poles. If may recall, we had spoken about how the system automatically becomes unstable if the poles on the fictitious axis of the s-plane coincide. It's okay if you can't recall. We may simply verify it using simulation.

As a result, in this instance both proportional and integral control are required. The proportional control aims to maintain system stability whereas the integral control eliminates steady state error.

Derivative Management

The rate of change of the error signal is affected by the derivative control, or put another way, the control action being proportional to the rate of change of an error signal. Let's use an illustration to clarify inFigure 6.11. Take into account the plant using the proportionate control shown below.

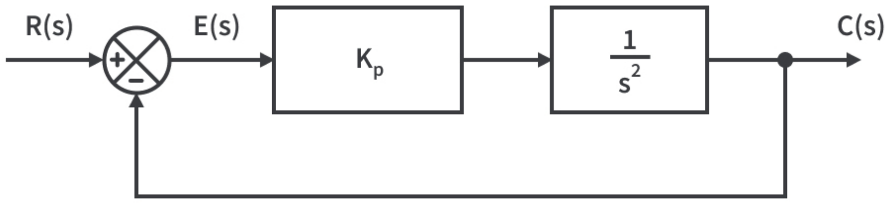


Figure 6.11: Illustrates the aforementioned system's closed loop transfer function.

$$\frac{C(s)}{R(s)} = \frac{K_p}{s^2+K_p}$$

This system is totally oscillatory regardless of the value of Kp since it is only moderately stable (has poles just on imaginary axis) for just a step input. Let's simulate and test with Kp = 10.

The schematic block,

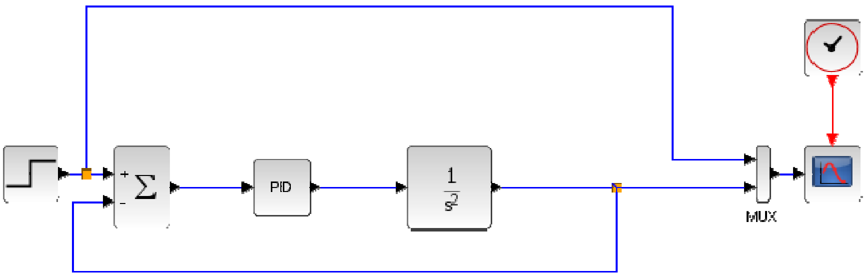


Figure 6.12. This system is totally oscillatory regardless of the value of Kp since it is only moderately stable (has poles just on imaginary axis) for just a step input.

The response,

This exhibits undamped oscillations, as can be seen. Our controller must dampen these oscillations in some way, and the derivative controller does this.

We'll apply derivative control to the prior illustration (Figure 6.12).

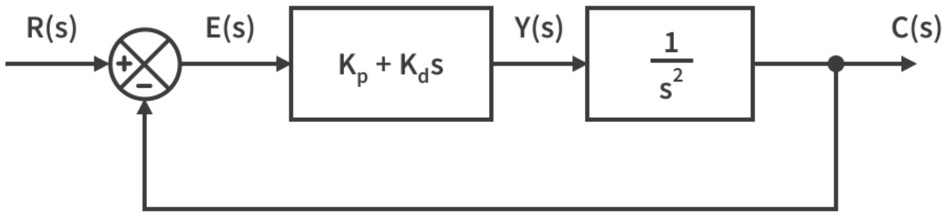


Figure 6.12 derivative control

The output of the controller is

$$Y(s) = (K_p + K_d s)\,E(s)$$

and taking the inverse Laplace transform of it,

$$y(t) = K_p e(t) + K_d \frac{de(t)}{dt}$$

As we discussed, the control action now depends on the rate of change of error.

The closed loop transfer function of the system now becomes

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{s^2 + K_d s + K_p}$$

Now this is always stable given that K_d and K_p are greater than 0. For simulation, we shall take $K_d = 5$ and $K_p = 10$.

This gives,

$$\xi = \frac{K_d}{2\sqrt{K_p}} = \frac{5}{2\sqrt{10}} = 0.79$$

Therefore, we should expect to witness a tiny overshoot since the system is slightly underdamped. This comment backs up what we just spoke about. Even the derivative control does not operate on its own. Always utilize proportional control while using it (proportional – derivative control). This is due to the fact that a system's output will be zero if it has a continuous error, which is the opposite of what we want to happen. Then, in order to understand proportional, integral, and derivative control (all of them combined), we will take an unstable system and attempt to force it to behave as we want.

$$G(s) = \frac{1}{s^2 - 10}$$

As one of its poles,

$$s = +\sqrt{10}$$

System instability lies inside the right half of the s-plane inFigure 6.13. Throughout this example, we will refer to the unit step input. Let's try stabilizing it with simply proportional control to get the required output first.

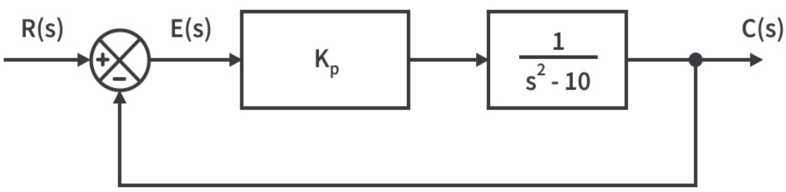


Figure 6.13: Illustrates the System instability lies inside the right half of the s-plane.

The closed loop transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{K_p}{s^2 - 10 + K_p}$$

This system can be brought from being unstable to being marginally stable if we make $K_p > 10$. So we choose $K_p = 20$ and check by simulating it inFigure 6.14. The block diagram,

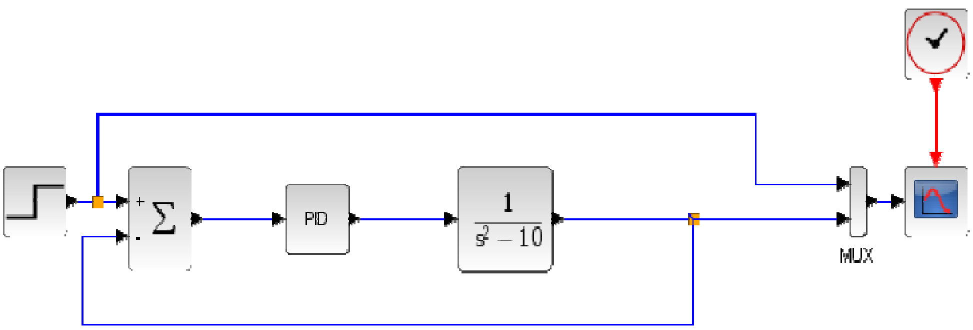


Figure 6.14: Illustrates this system can be brought from being unstable to being marginally stable.

The response,

Clearly, proportional control alone won't be enough to stabilize the system. Derivative control will be added since they need to dampen the oscillations in this situation inFigure 7.13.

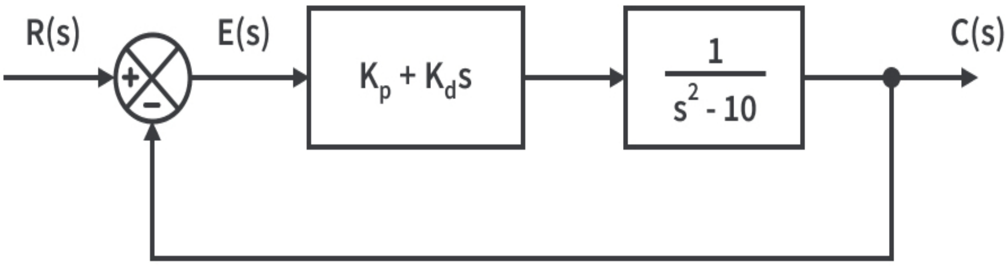


Figure 6.15: Illustrates the derivative control will be added since they need to dampen the oscillations in this situation.

The closed loop transfer function now becomes,

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{s^2 + K_d s + K_p - 10}$$

This will be stable for $K_p > 10$ and $K_d > 0$, so we arbitrarily choose $K_p = 20$ and $K_d = 10$ and check the response (Figure 6.16).

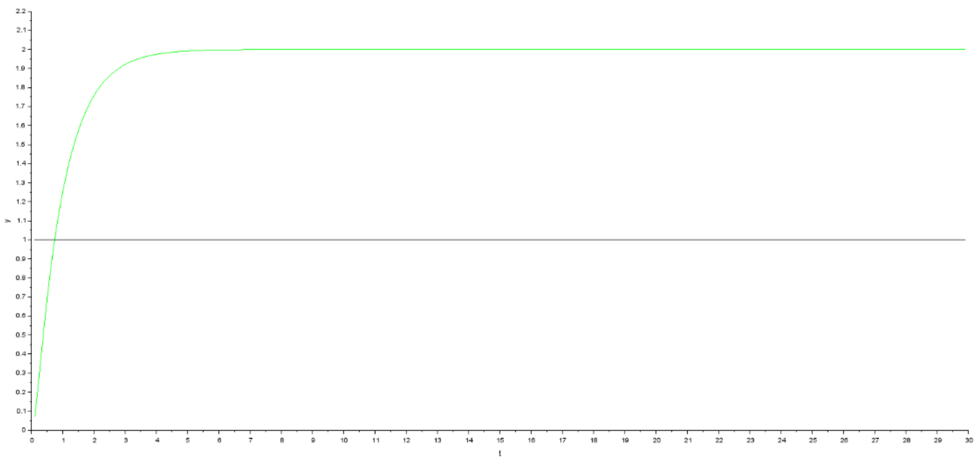


Figure 6.16 closed loop transfer function

Although the system currently seems to be stable, there appears to be a significant steady state error. Designers use the integral controller to lower the steady state error. Let's now add the integral controller (Figure 6.17):

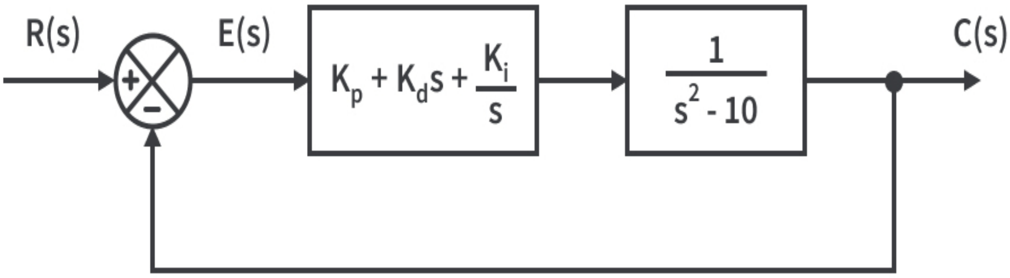


Figure 6.17 steady state error

The closed loop transfer function will now be,

$$\frac{C(s)}{R(s)} = \frac{K_p s^2 + K_p s + K_i}{s^3 + K_d s^2 + (K_p - 10)s + K_i}$$

For this system to be stable,

s^3	1	$K_p - 10$
s^2	K_d	K_i
s	$K_d(K_p - 10) - K_i$	
1	K_i	

$$K_d > 0, K_i > 0$$

$$K_d(K_p - 10) > K_i$$

We choose Kp = 20, Kd = 10, and Ki = 5 at random, which stabilizes the system (10 * (20 - 10) > 5).

As we can see, the steady state error is removed in this case, and we used PID control to get a stable response. The parameters Kp, Kd, and Ki may be correctly tuned to provide even better tracking. One can still get a respectable response by modifying these settings by trial and error, even though there are a few particular strategies for doing so that are beyond the scope of this instructional series. Proportional control came first, and then we learned about integral control. We sought to resist disruption by using both of them. After learning about derivative control, we utilized it to dampen the response together with a proportional control. Later, we stabilized an unstable system by making overall steady state error zero and bringing its response near to the intended one using proportional, derivative, and integral control.

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CHAPTER 7

COMPENSATORS AND CONTROLLERS

Dr. Sreenivasappa B V, Professor
Department of Electronics and Communication Engineering, Presidency University, Bangalore, India
Email Id- sreenivasappabv@presidencyuniversity.in

In order to obtain the required output from the primary plant system, controllers or compensators essentially act alongside it. Is it necessary when we can just build the primary system to provide the desired results? It's because a system's characteristics might alter as a result of lengthy usage (aging), and the output and time response requirements that result may not be what you were hoping for. The system should now be modified so that it responds to our requests once more, although that is not always achievable. Once a system has been built and established, it might not be possible to change it. Therefore, having controllers or compensators with adjustable/tunable characteristics in addition to the primary system aids in minimizing the changes that the main system experiences as a result of variables like aging, outside disturbances, etc. The duties of controllers or compensators might include stabilizing the output, reducing overshoot, minimizing settling time, and keeping it within a specified range. Below is a generic block diagram of a closed loop system with a controller or compensator[1]–[7].We will quickly review the time response parameters we learned in lesson 2.4 before moving on. A general closed loop second order system is explained by,

$$T(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

These time response standards provide a description of this system's response:

- Rise Time

$$t_r = \frac{\pi - cos^{-1}\zeta}{w_n\sqrt{(1-\zeta^2)}}$$

- Delay Time

$$t_d = \frac{1 + 0.7\zeta}{w_n}$$

- Peak Time

$$t_p = \frac{\pi}{w_d} = \frac{\pi}{w_n\sqrt{1-\zeta^2}}$$

- Percentage Overshoot

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} * 100\%$$

- Settling Time

$$t_s = \frac{4}{\zeta w_n}(2\%\ tolerance)$$

Almost all of these are required in order to evaluate the effectiveness of a control system. But this only applies to second order systems, not all of which we encounter in the actual world.

Let's say we have a higher order system (more than two poles). Each pole or zero contributes to the system's reaction in some way, although certain poles tend to have a significantly bigger impact on the system's response than others. These poles are known as the dominating poles. The system is then approximated by disregarding poles other than the dominant ones and employing the same temporal response criteria as before (Figure 7.1).

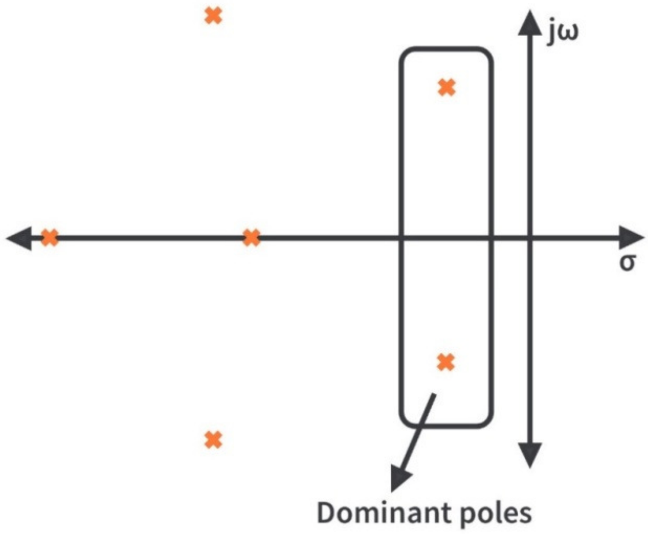


Figure 7.1 illustration of a higher order system as well as its prominent poles

As we continue with this course, the idea of dominating poles will become more evident, so hang on tight. Let's now examine systems that also have a zero or a pole in addition to their dominating poles. Prior to that, we'll use the transfer function of the series RLC circuit for this study, as illustrated below in Figure 7.2.

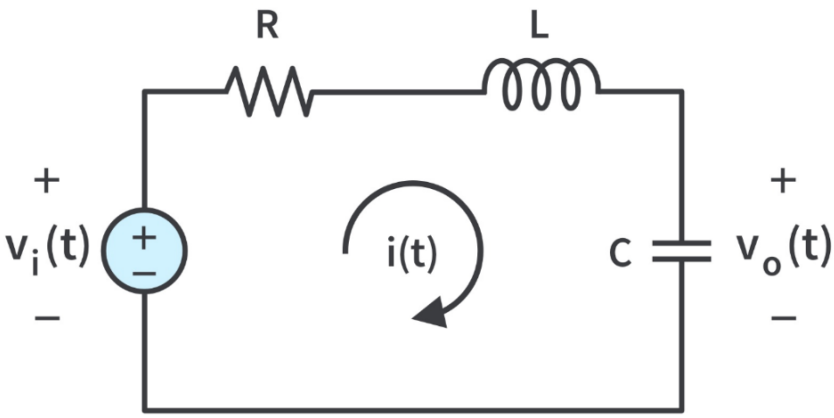


Figure 7.2: Illustrates the circuit diagram of RLC.

$$\frac{V_o(s)}{V_i(s)} = G(s) = \frac{\frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Now, the response after adding a zero at s = -a becomes,

$$G_1(s) = G(s) * G_c(s) = \frac{1 - \frac{s}{a}}{s^2 + s + 1}$$

We have taken the values of a to be 0.5, 1, and 10 and simulated it to see its effect on the response of the system using Scilab.

When a pole or zero is added at different points, its impact on the reaction decreases as the value of "a" rises, i.e., as the pole or zero travels further from the hypothetical axis. Visitors can see from this study that the response is most influenced by a value of 0.5 and the system is least affected by a value of 10. We may also conclude that the dominating pole notion makes it simpler to analyze systems of higher order. The higher order systems often have the dominating poles arranged in conjugate pairs. The real part of the other poles in the system must be more than five times overall real part of the dominant pole combination in order to consider a pair of poles to be dominant. One may change the response of the system to the desired one by adding a zero or a pole. For instance, by looking at the plots, we can determine that adding a zero to the left side of the s-plane would be effective if we were to speed up the response. Similarly, if they need to slow down the system, we may do so by adding a pole or a zero to the left or right halves of the s-plane, respectively; however, in the former case, one can see an undershoot as shown in the plots. By observing the requirements in this manner, one may modify the system's reaction by adding a pole or a zero. If may remember, they also covered the root-locus plot in the last lesson while talking about the effects of adding a pole or a zero on stability. As a result, we may conclude that the addition of a pole or even a zero can alter the system response and have an influence on the stability of the system, serving as the foundation for controller design in control systems.

Root Locus Plot

Consider the system with an open-loop transfer function Figure 7.3.

$$G(s) = \frac{K}{s(s+7)(s+11)}$$

Where K is positive

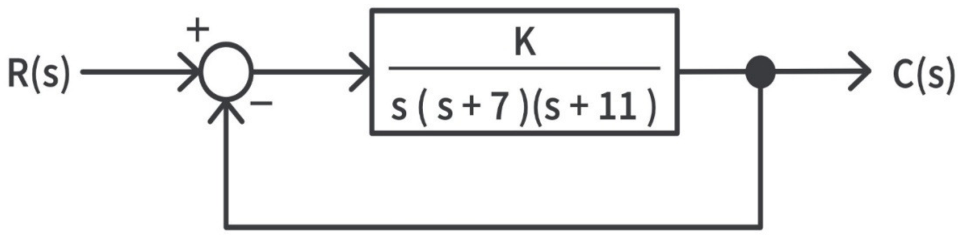


Figure 7.3: Illustrates to consider the system with an open-loop transfer function.

Its closed-loop transfer function is

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{K}{s^3+18s^2+77s+K}$$

And the characteristic equation

$$q(s) = s^3+18s^2+77s+K$$

The constant K here may just represent some constant, but in actual systems, it may be an unknown parameter or a variable parameter and, as we can see in this example, this unknown parameter affects the characteristic equation and in turn affects the location of poles of the system.

To check this, let's assign a few values to K and then see how the location of poles varies.

The first is the Evans criteria, which serve as the foundation for the root locus approach (Evan was its creator). Circumstances, Evans Think about the following usually closed-loop system (Figure 7.4).

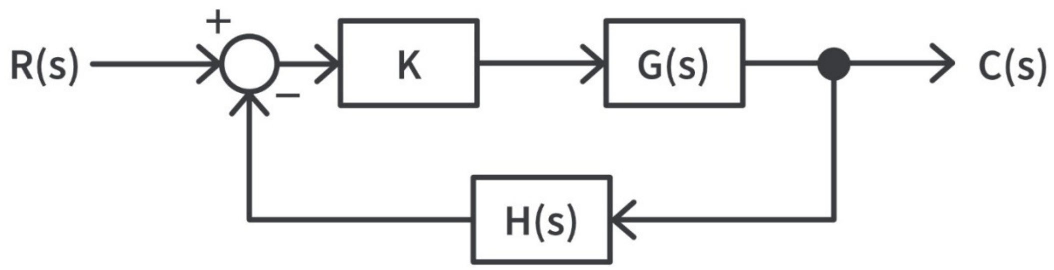


Figure 7.4 closed-loop system

The closed-loop transfer function of the system is given by

$$T(s) = \frac{KG(s)}{1+KG(s)H(s)}$$

As we considered earlier, K is the variable parameter, and we will assume K can vary from

$$0 \le K \le \infty$$

The characteristic equation of the system is given by

$$q(s) = 1+KG(s)H(s) = 0$$

$$KG(s)H(s) = -1$$

As s is a complex variable, we can rewrite the above equation as

$$|KG(s)H(s)| \angle(KG(s)H(s)) = -1+j0 = 1\angle(\pm(2q+1)180^\circ); q = 0,1,2,\cdots$$

This means that for

$$KG(s)H(s)$$

It must create an angle that is odd multiples of 180° in order for it to be -1 (to be on the true negative axis). They get Evans conditions as well as the Magnitude Criterion by comparing the two sides of the aforementioned equation,

$$|KG(s)H(s)| = 1$$

The Angle Criterion is used in situations when the magnitude of the open-loop transfer function must be 1.

$$\angle(KG(s)H(s)) = \pm(2q+1)180^\circ; q = 0, 1, 2, ...$$

Where the odd multiple of 180° is the angle of the complex open-loop transfer function for each and every point on the root locus.

Flow Chart for Signals

An algebraic equation is graphically represented in a signal flow graph. Let's talk about the fundamental ideas behind signal flow graphs in this chapter and also discover how to create one.

Basic Signal Flow Graph Elements

The fundamental components of a signal flow graph are nodes and branches.

Node A node is a point that symbolizes a signal or a variable. Three different node kinds exist.

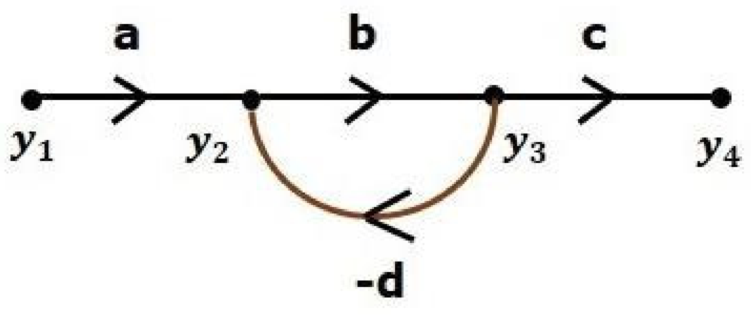
Output node, mixed node, and input node.

A node with only outgoing branches is indeed an input node.

An output node only contains incoming branches is known as an output node.

A node with both incoming and outgoing branches was referred to as a mixed node.

Example



- The **nodes** present in this signal flow graph are y_1, y_2, y_3 and y_4.
- y_1 and y_4 are the **input node** and **output node** respectively.
- y_2 and y_3 are **mixed nodes**.

To locate these nodes, let's consider the following signal flow graph.

Branch A branch connects two nodes on a line. It has orientation as well as gain. For instance, the signal flow graph above has four branches. The gains on these branches are a, b, c, and -d.

Building a Signal Flow Graph

Let's create a signal flow graph by taking into account the following algebraic equations.

$$y_2 = a_{12}y_1 + a_{42}y_4$$

$$y_3 = a_{23}y_2 + a_{53}y_5$$

$$y_4 = a_{34}y_3$$

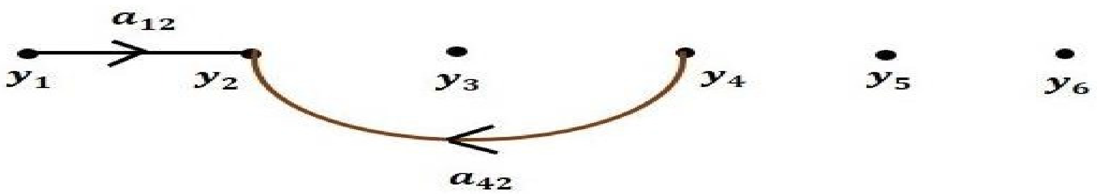
$$y_5 = a_{45}y_4 + a_{35}y_3$$

$$y_6 = a_{56}y_5$$

There will be six nodes (V1, V2, V3, V4, V5 and y6) and eight branches in this signal flow graph. The gains of the branches are a12, a23, a34, a45, a56, a42, a53 and a35.

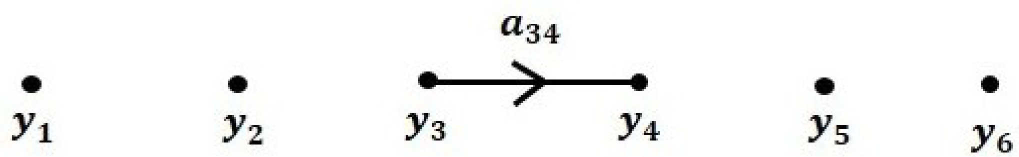
To get the overall signal flow graph, draw the signal flow graph for each equation, then combine all these signal flow graphs and then follow the steps given below -

Step 1 figure. Signal flow graph for y2 = a13Y1 +04294 is shown in the following

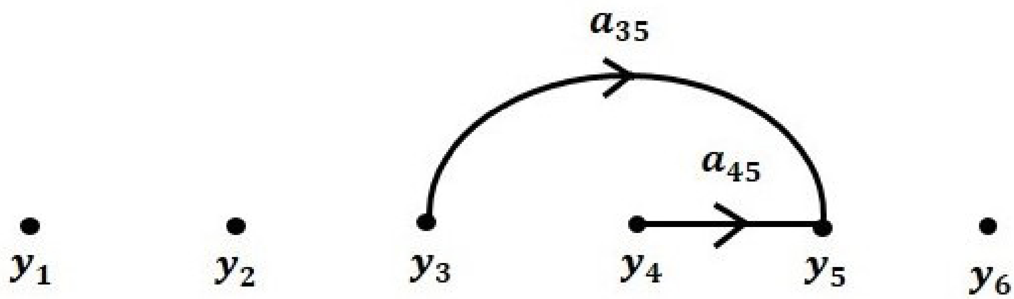


Step 2 − Signal flow graph for $y_3 = a_{23}y_2 + a_{53}y_5$ is shown in the following figure.

Step 3 − Signal flow graph for $y_4 = a_{34}y_3$ is shown in the following figure.

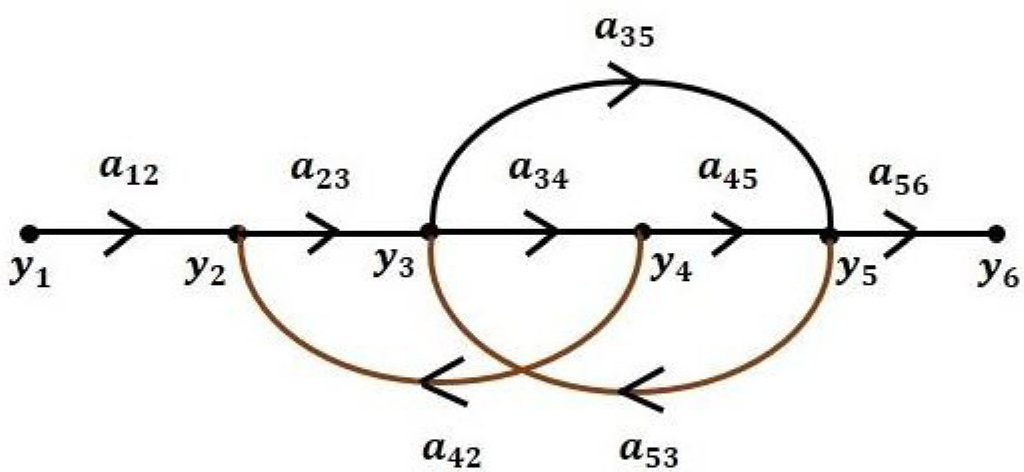


Step 4 – Signal flow graph for $y_5 = a_{45}y_4 + a_{35}y_3$ is shown in the following figure.



Step 5 – Signal flow graph for $y_6 = a_{56}y_5$ is shown in the following figure.

Step 6 – Signal flow graph of overall system is shown in the following figure.



Block diagrams are transformed into signal flow graphs.

To translate a block diagram itself into the corresponding signal flow graph, follow these steps. Create a signal flow graph with nodes for each of the block diagram's signals, variables, summing points, and takeoff points. Interpret the signal flow graph's branching as that of the block diagram's building blocks. Represent the gains of a branches inside the signal flow graph as the transfer functions within the blocks of the block diagram. According to the block diagram, interconnect the nodes. If there is a link between two nodes (but no block), therefore the gain of the branch should be represented as 1. For instance, between input and takeoff points, input and summing points, input and output, as well as between summing points.

Example

Let's create a signal flow graph using the following block diagram Figure 7.5.

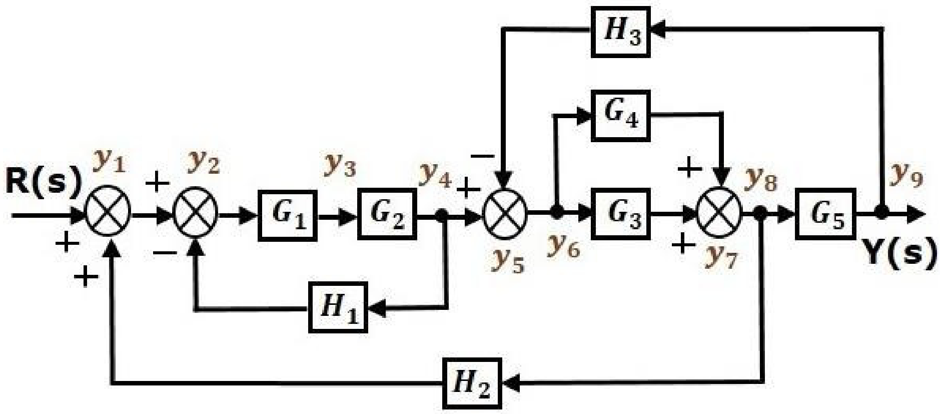


Figure 7.5: Illustrates the signal flow graph using the following block diagram.

Use the input node R(s) and output node C(s) of a signal flow graph to represent the input signal R(s) and output signal C(s) of a block diagram.

The block diagram labels the remaining nodes (y1 to y9) for convenience. Other than the input and the output nodes, there seem to be nine nodes. This results in one node again for variable between blocks G1 and G2, four nodes for the four take-off points, but also four nodes for the four summing points.The similar signal flow graph is shown in the following image in Figure 7.6.

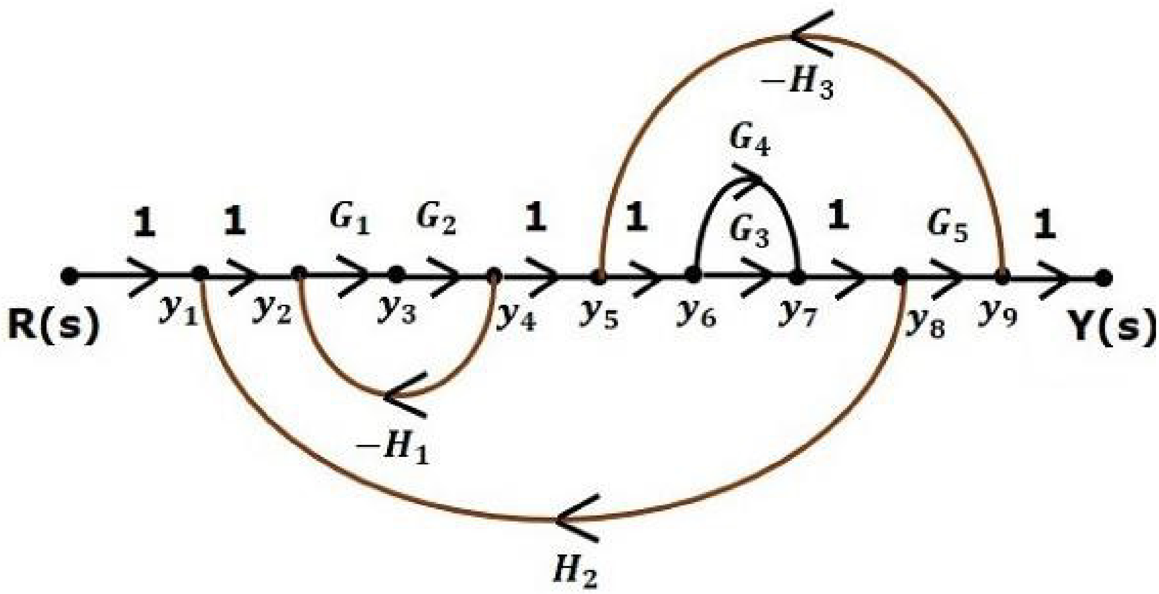


Figure 7.6: Illustrates the signal flow graph with their summing point.

Now, let's talk about the Mason's Gain Formula. Consider a signal flow graph with "N" forward pathways. A signal flow graph's gain between both the input and output nodes is nothing more than the system's transfer function. Mason's Gain Formula may be used to compute it.

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CHAPTER 8

MASON'S GAIN

Dr. Abhishek Kumar Sharma, Assistant Professor
Department of Computer Science and Engineering, Sanskriti University, Mathura, Uttar Pradesh, India
Email Id- abhishek.sharma@sanskriti.edu.in

Mason's gain formulais

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

C(s) is the output node where

The input node is R(s).

T is the gain between R(s) and C's transfer function (s)

Pi is the ith gain on a forward route.

Δ=1

(Total gain of each individual loop) + (Total gain of each of the two non-touching loops that might occur) + (Total gain of each of the three non-touching loops that could occur) +

By deleting the loops that touch the ith forward route, they can get I from[1]–[10]. To comprehend the fundamental language used here, have a look at the following signal flow graph (Figure 8.1).

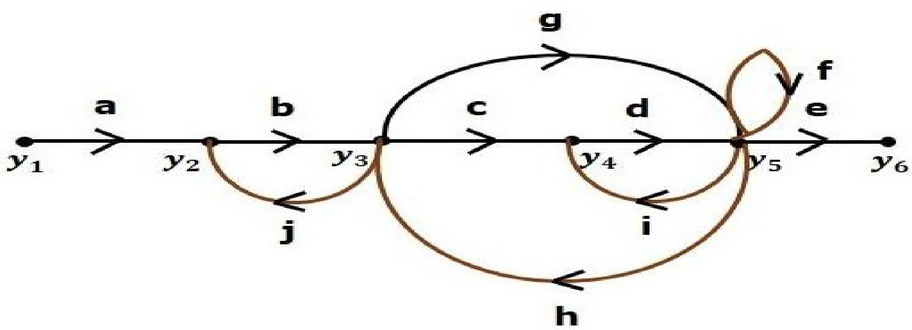


Figure 8.1 signal flow graph

Path

It is a traversal of branches from one node to any other node in the direction of branch arrows. It should not traverse any node more than once.

Examples - y2 → Y3 → Y4 → y5 and y5 → Y3 → Y2

Forward Path

The path that exists from the input node to the output node is known as forward path.

Examples - y1 → Y2 → Y3 → Y4 → Y5 → Y6 and y1 → Y2 → Y3 → Y5 → Y6 ·

Forward Path Gain

It is obtained by calculating the product of all branch gains of the forward path.

Examples - abcde is the forward path gain of y1 → Y2 → Y3 → Y4 → Y5 → Y6 and abge is the forward path gain of y1 y2 → Y3 → Y5 → Y6 ·

Loop

A loop is a route that originates at one node and terminates at that same node. It is thus a closed route.

Examples - y2 y3 → Y2 and y3 → Y5 → Y3.

Loop Gain

It is obtained by calculating the product of all branch gains of a loop.

Examples - bj is the loop gain of y2 → Y3 → y2 and gh is the loop gain of

Y3 → Y5 → Y3.

Non-touching Loops

These are the loops, which should not have any common node.

Examples - The loops, y2 → Y3 → Y2 and y4 → Y5 → Y4 are non-touching.

CalculationofTransferFunctionusingMason'sGainFormula

Let's use the same signal flow graph to determine the transfer function (Figure 8.2).

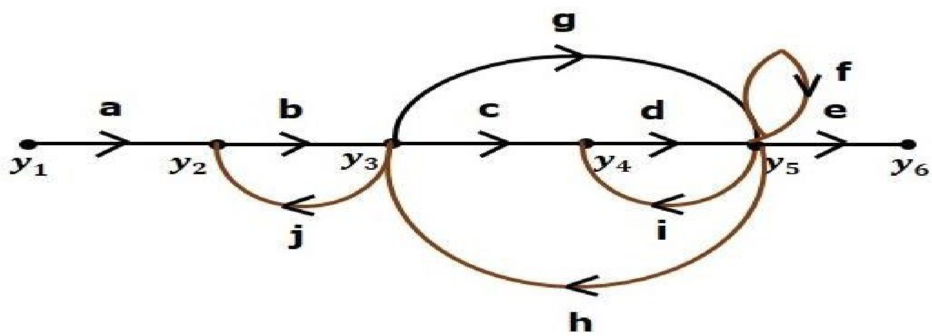


Figure 8.2 signal flow graph

N = 2, the number of forward pathways.

The first forward-looking route is y1y2y3y4y5y6.

Gain on the first forward route, p1=abcde

This is the second forward path: y1y2y3y5y6.

Gain on the second forward route, p2=abge.

Numberofindividualloops,L =5.

Loops are Y2 → Y3 → Y2, Y3 → Y5 → Y3, Y3 → Y4 → Y5 → Y3. -

Y4 → Y5 → Y4 and y5 → Y5.

Loop gains are - l□ = bj, l□ = gh, l3 = cdh, l = di and l5 = f.

There are two loops that are not touching.

The first non-touching loop pair is y2y3y2, followed by y4y5y4.

First non-touching loops' gain product is l1l4=bjdi.

The second non-touching loop pairing is y2y3y2, followed by y5y5.

L1L5=bjf is the gain product of the second pair of non-touching loops.

Throughout this signal flow graph, there aren't any non-touching loops with a higher number (greater than two).

We are aware of (Figure 8.3),

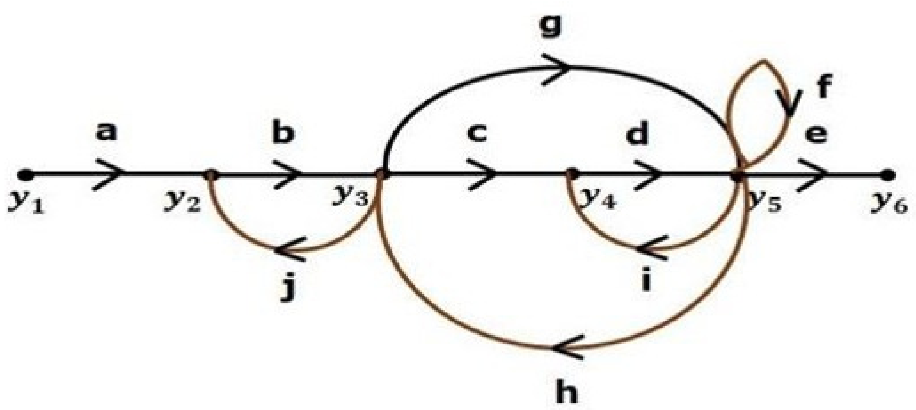


Figure 8.3 signal flow graph

Number of forward paths, N = 2.

First forward path is y1 y2 y3 y4→ Y5 → Y6. -

First forward path gain, P□ = abcde.

Second forward path is y1 y2 y3 → Y5Y6. -

Second forward path gain, p2 = abge.

Number of individual loops, L = 5.

We know,

▲ = 1 - (sum of all individual loop gains) +(sum of gain products of all possible two nontouching loops) -(sum of gain products of all possible three nontouching loops) +...Substitute the values in the above equation, A = 1 - (bj+gh + cdh +di+f) + (bjdi + bjf) — (0)⇒ ▲ = 1 – (bj +gh +cdh + di + f) + bjdi +bjfSo, A_1 = 1.

Similarly, A2 = 1. Since, no loop which is non-touching to the second forward path.

Substitute, N = 2 in Mason's gain formula

$y_4 \rightarrow y_5 \rightarrow y_4$ and $y_5 \rightarrow y_5$.

T= C(s) R(s) ΣAPIΔ Δ

T= C(s) R(s) P1A1 + P2A2 A

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Loops are - Y2Y3 → Y2, Y3Y5Y3, Y3Y4Y5 → Y3,

Loop gains are - l□ = bj, l□ = gh, l3 = cdh, 14 = di and l5 = f.

Number of two non-touching loops = 2.

First non-touching loops pair is y2 y3 →Y2, Y4 → Y5 → Y4.

Gain product of first non-touching loops pair, l1l4 = bjdi

Second non-touching loops pair is - y2 y3 →→Y2, Y5 → Y5.

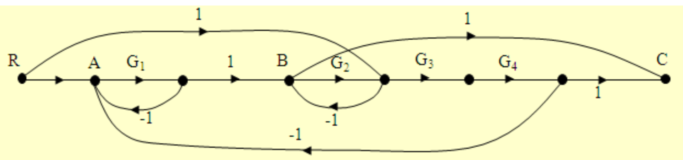
Gain product of second non-touching loops pair is - l□l = bjf

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

Therefore, the function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Example-1:DeterminethetransferfunctionC(s)/R(s).



$M_1 = G_1G_2G_3G_4 \qquad \Delta_1 = 1$

$M_2 = G_1 \qquad \Delta_2 = 1$

$M_3 = G_3G_4 \qquad \Delta_3 = 1 + G_1$

$M_4 = -1 \qquad \Delta_4 = 1 + G_1$

$M_5 = -G_3G_4G_1 \qquad \Delta_5 = 1$

$$\Delta = 1 - (-G_1 - G_2 - G_1G_2G_3G_4) + G_1G_2 \ = \ 1 + G_1 + G_2 + G_1G_2G_3G_4 + G_1G_2$$

a) $$\frac{C}{M} = \frac{G_1G_2G_3G_4 + G_1 + G_3G_4(1+G_1) - (1)(1+G_1) - G_1G_3G_4}{1 - (-G_1 - G_2 - G_1G_2G_3G_4) + G_1G_2}$$

Analysis of time and response

Both the time domain as well as the frequency domain may be used to examine the response of a control systems. Later chapters will include frequency response analysis of control systems. Let's now talk about the control systems' time response analysis.

Time Reaction

The term "time response of the control system" refers to how a control system responds to an input while its output changes over time. There are two components to the time response.

Instantaneous reaction

Stable state reaction

The following graphic displays the control system's reaction in the temporal domain in Figure 8.4.

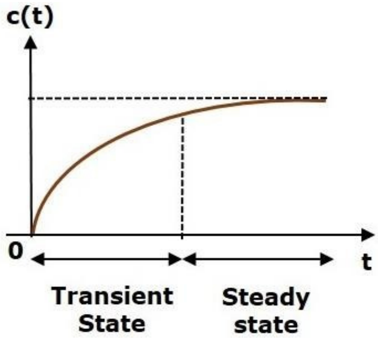


Figure 8.4: Illustrates both the transient and the steady states.

Here, both the transient and the steady states are indicated in the figure. The responses corresponding to these states are known as transient and steady state responses.

Mathematically, we can write the time response c(t) as

c(t) = Ctr (t) + Css (t)

Where,

c_{tr}(t)isthetransientresponse

c_{SS}(t)isthesteadystateresponse

Temporary Response

It takes a specific amount of time for output to achieve steady state after applying input to the control system. Therefore, until it reaches a steady state, the output will be in a transitory condition. Transitory reaction is the term for the control system's response while it is in a transient condition.

If foot' is big, the transient reaction will be nil. This value of "t" should be infinite in theory, but in practice it is five times constant.

In mathematics, it may be expressed as,

$$\lim_{t \to \infty} c_{tr}(t) = 0$$

Response in a stable state

The steady state response is the portion of the temporal response that persists even after the transient response achieves zero value for large values of't'. This implies that even at steady state, the transient reaction will be zero.

Example

Let's determine the transient and steady state components of the control system's temporal

$$c(t) = 10 + 5e^{-t}$$

response.

Here,thesecondterm $5e^{-t}$ willbezeroastdenotesinfinity.So,thisisthetransientterm. And even as t approaches infinity, the first term (10), stays the same. Thus, this word refers to a stable condition. Common test signals

Impulse, step, ramp, and parabolic signals are the typical test signals. Utilizing the output's temporal response, those signals are utilized to evaluate the performance of the management systems.

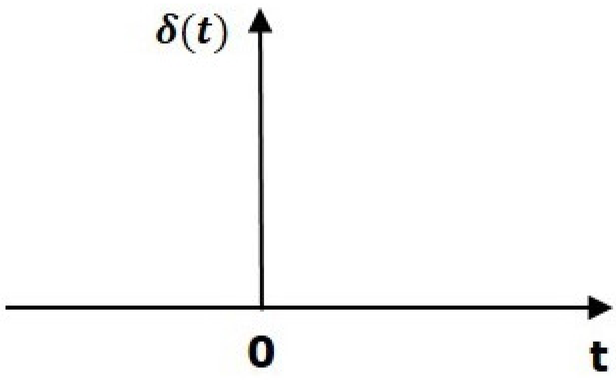
Signal with Unit Impulse

The definition of a unit impulse signal t is,

$$\delta(t) = 0 \text{ for } t \neq 0$$

$$\text{and } \int_{0^-}^{0^+} \delta(t)dt = 1$$

The following figure shows unit impulse signal,



Therefore, the unit impulse signal only occurs when t is equal to 0. This signal's area at tiny time intervals around t' equals zero and is one. For all other values of "t," the unit impulse signal has a value of 0.

Step Unit Signal

The definition of a unit step signal, u(t), is,

$$u(t) = 1; t \geq 0$$

$$= 0; t < 0$$

FollowingFigure 8.5showsunitstepsignal.

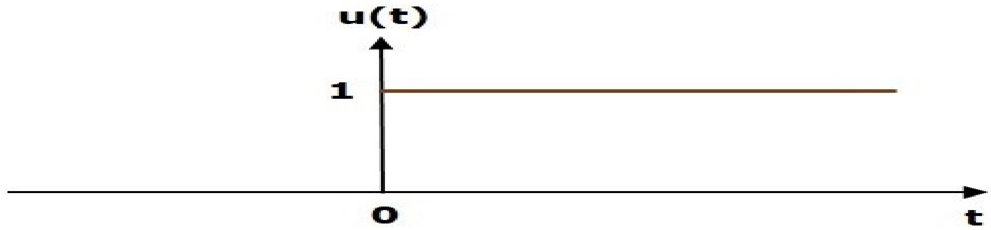


Figure 8.5: Illustrates the unitstepsignal.

For any and all positive values of "t," including zero, the unit step signal is present. And throughout this time, its value is 1. For any and all negative values of't, the unit step signal has a value of zero'.

UnitRampSignal

Aunitrampsignal,r(t) isdefinedas,

$$r(t) = t; t \geq 0$$

$$= 0; t < 0$$

We can write unit ramp signal, r(t) in terms of unit step signal, u(t) as

r(t) = tu(t)

Following Figure 8.6 shows unit ramp signal.

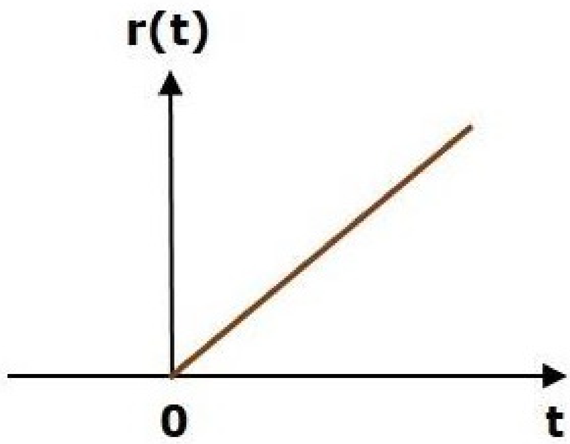


Figure 8.6: Illustrates the unit ramp signal.

For any and all positive values of "t," including zero, the unit ramp signal was present. During this time, its value grows linearly with respect to t. For all negative values of "t," the unit ramp signal has a value of 0.

Parabolic Unit Signal

$$p(t) = \frac{t^2}{2}; t \geq 0$$

$$= 0; t < 0$$

The definition of a unit parabolic signal, p(t), is, We can write unit parabolic signal, p(t) in terms of the unit step signal, u(t) as,

$$p(t) = \frac{t^2}{2}u(t)$$

The following Figure 8.7 shows the unit parabolic signal.

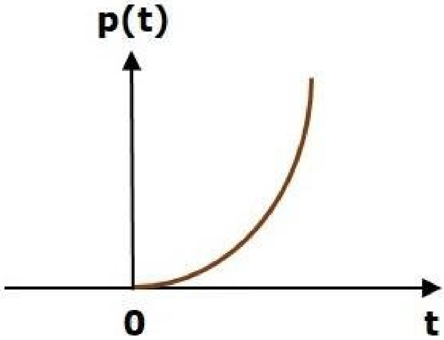


Figure 8.7: Illustrates the unit parabolic signal with respect to time.

Therefore, all positive values of "t," including zero, exhibit the unit parabolic signal. Additionally, throughout this time, its value rises non-linearly with respect to t. For all negative values of "t," the unit parabolic signal has a value of "0."

Let's talk about the first order system's temporal response in the chapter. Take a look at the closed loop control system block diagram below.

We know that the transfer function of the closed loop control system has unity negative feedback as,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Substitute, G(s) = in the above equation.

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1+\frac{1}{sT}} = \frac{1}{sT+1}$$

The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the first order system.

We can re-write the above equation as

$$C(s) = \left(\frac{1}{sT+1}\right)R(s)$$

Where,

C(s) is the Laplace transform of the output signal c(t),

R(s) is the Laplace transform of the input signal r(t), and

T is the time constant. Tis

Follow these steps to get the response (output) of the first order system in the time domain.

Take the Laplace transform of the input signal r(t).

Consider the equation, C'(s) = ST+1 R(s)

Substitute R(s) value in the above equation.

Do partial fractions of C(s) if required.

Apply inverse Laplace transform to C'(s).

First-order system's impulse response

Think of the unit impulse signal as the primary order system's input.

So, r(t)=δ(t) (t)

Put the Laplace transform to use on both sides. R(s) =1.

Consider the equation,

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

Substitute, R(s) = 1 in the above equation.

$$C(s) = \left(\frac{1}{sT+1}\right)(1) = \frac{1}{sT+1}$$

Transform the aforementioned equation into one of the Laplace transforms' standard forms.

$$C(s) = \frac{1}{T\left(s+\frac{1}{T}\right)} \Rightarrow C(s) = \frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$$

ApplyingInverseLaplaceTransformonboththesides,

$$c(t) = \frac{1}{T} e^{\left(-\frac{t}{T}\right)} u(t)$$

$$c(t) = \frac{1}{T} e^{\left(-\frac{t}{T}\right)} u(t)$$

TheunitimpulseresponseisshowninthefollowingFigure in 8.8.

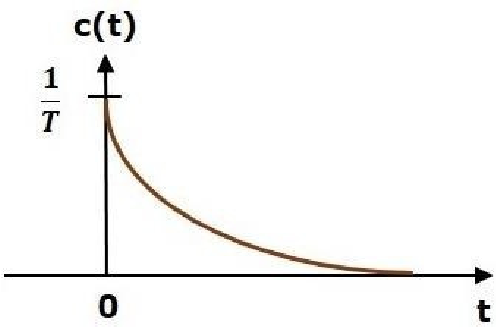


Figure 8.8: Illustrates the unit impulse response.

For positive values of't,' the unit impulse response, c(t), is indeed an exponentially decaying signal, and also for negative values of 't,' it is zero.

First Order Step Response System

Think of the unit step signal as the initial order system's input. So, r(t)=u (t).

R(s) = S

Consider the equation,

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

Substitute, R(s) = in the above equation,

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s}\right) = \frac{1}{s\left(sT+1\right)}$$

Do partial fractions of C(s),

$$C(s) = \frac{1}{s\left(sT+1\right)} = \frac{A}{s} + \frac{B}{sT+1}$$

$$\Rightarrow \frac{1}{s\left(sT+1\right)} = \frac{A\left(sT+1\right)+Bs}{s\left(sT+1\right)}$$

The denominator phrase is the same on both sides. Consequently, they will cancel each other out. So, compare the terms in the numerator.

1=A(sT+1)+Bs

A = 1 is obtained by equating the constant terms on the both sides. A = 1 should be substituted, and the coefficients of the s terms on both sides should be equal.

0=T+B \s⇒B=−T

Replace C with a partial fraction expansion where A = 1 and B = T. (s).

$$C(s) = \frac{1}{s} - \frac{T}{sT+1} = \frac{1}{s} - \frac{T}{T\left(s+\frac{1}{T}\right)}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

Use inverse Laplace transformation from both angles.

The**unitstepresponse**,c(t)hasboth thetransientandthesteadystateterms.

$$c(t) = \left(1 - e^{-\left(\frac{t}{T}\right)}\right) u(t)$$

Thetransienttermintheunitstepresponse is-

$$c_{tr}(t) = -e^{-\left(\frac{t}{T}\right)}u(t)$$

The unit step response's steady state term is: The unit step reaction is shown in the accompanying Figure 8.9.

$$c_{ss}(t) = u(t$$

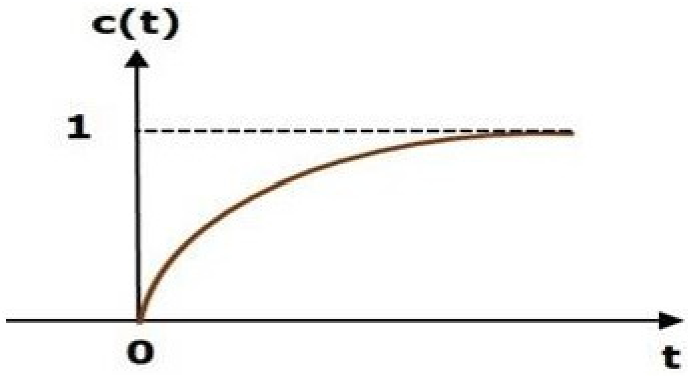


Figure 8.9: Illustrates the unit step reaction is shown in the accompanying.

At t = 0 and for all other negative values of t, the unit step response, c(t), has a value of zero. It starts out at 0 and increases steadily until it reaches one in a steady state. Therefore, the steady state value is influenced by the size of the input.

System First Order Ramp Response

Think of the unit ramp signal as the initial order system's input.

Hence, r(t)=t u (t)

Laplace transform should be used on both sides.

$$R(s) = \frac{1}{s^2}$$

Consider the equation, $C(s) = \left(\frac{1}{sT+1}\right)R(s)$

Substitute, $R(s) = \frac{1}{s^2}$ in the above equation.

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s^2}\right) = \frac{1}{s^2(sT+1)}$$

Do partial fraction of C(s),

$$C(s) = \frac{1}{s^2(sT+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{sT+1}$$

$$\Rightarrow \frac{1}{s^2(sT+1)} = \frac{A(sT+1) + Bs(sT+1) + Cs^2}{s^2(sT+1)}$$

The denominator phrase is the same on both sides. Consequently, they will cancel each other

$$1 = A(sT+1) + Bs(sT+1) + Cs^2$$

out. So, compare the terms in the numerator.

By equating the constant terms on both sides, we arrive at A = 1. A should be changed to 1, and the coefficient of the s terms should be same on both sides.

0=T+B⇒B=−T

Replacing B with T in a similar manner will equalize the coefficient of the s2 terms on both sides. You'll get C=T2

Replace C=T2 in the partial fraction expansion of C with A=1, B=T, and (s).

$$C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{sT+1} = \frac{1}{s^2} - \frac{T}{s} + \frac{T^2}{T\left(s+\frac{1}{T}\right)}$$

$$\Rightarrow C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+\frac{1}{T}}$$

Use inverse Laplace transformation from both angles.

$$c(t) = \left(t - T + Te^{-\left(\frac{t}{T}\right)}\right)u(t)$$

Both the transient as well as the steady state elements are included in the unit ramp response, c(t). The unit ramp response's transient term is.

$$c_{tr}(t) = Te^{-\left(\frac{t}{T}\right)}u(t)$$

Thesteadystatetermintheunitramp responseis

$$c_{ss}(t) = (t-T)u(t)$$

TheFigure 8.10belowistheunit rampresponse:

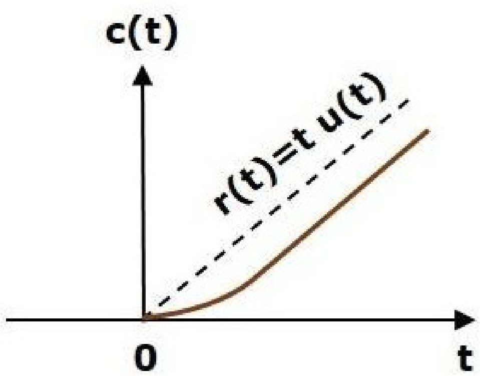


Figure 8.10: Illustrates thefigurebelowistheunit rampresponse.

For all positive values of t, the unit ramp response, c(t), follows the unit ramp input signal. However, the output signal differs by T units from the input signal.

ParabolicResponseofFirstOrderSystem

Consider of the units parabolic signal as the first order system's input.

So,

$$r(t) = \frac{t^2}{2}u(t)$$

Apply Laplace transform on both the sides.

$$R(s) = \frac{1}{s^3}$$

Consider the equation,

$$C(s) = \left(\frac{1}{sT+1}\right)R(s)$$

Substitute R(s) = in the above equation,

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s^3}\right) = \frac{1}{s^3(sT+1)}$$

Do partial fractions of C'(s),

$$C(s) = \frac{1}{s^3(sT+1)} = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{D}{sT+1}$$

After simplifying, you will get the values of A, B, C and D as 1, -T, T^2 and - T^3 respectively. Substitute these values in the above partial fraction expansion of C(s),

$$C(s) = \frac{1}{s^3} - \frac{T}{s^2} + \frac{T^2}{s} - \frac{T^3}{sT+1} \Rightarrow C(s) = \frac{1}{s^3} - \frac{T}{s^2} + \frac{T^2}{s} - \frac{T^2}{s+\frac{1}{T}}$$

ApplyinverseLaplacetransformonboththe sides.

$$c(t) = \left(\frac{t^2}{2} - Tt + T^2 - T^2e^{-\left(\frac{t}{T}\right)}\right)u(t)$$

Both the transient and steady state components are included in the unit parabolic response,

$$C_{tr}(t) = -T^2e^{-\left(\frac{t}{T}\right)}u(t)$$

c(t). The unit parabolic response's transient term is.

Within the unit parabolic response, the steady state term is

As a result of these reactions, we may infer that first order control systems with ramp and parabolic inputs are unstable since they continue to grow indefinitely. Due to the limited output of these responses, first order control systems are stable given impulse and step inputs.

But there is no steady state name for the impulse response. In order to analyze control systems based on their responses, the step signal is thus often utilized in the time domain.

Let's talk about the second order system's temporal response in this chapter. Take a look at the closed loop control system block diagram below. A unity negative feedback is attached to an open loop transfer function, n2 / s(s+2n), in this example.

$$C_{ss}(t) = \left(\frac{t^2}{2} - Tt + T^2\right)u(t)$$

We know that the transfer function of the closed loop control system having unity negative feedback as,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

The denominator term's power of "s" is two. As a result, the system is referred to as a second order system since the aforementioned transfer function is from the second order.

Thecharacteristicequationis-

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$

Substitute, $G(s) = \frac{\omega_n^2}{s(s+2\delta\omega_n)}$ in the above equation.

$$\frac{C(s)}{R(s)} = \frac{\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)}{1+\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

The roots of characteristic equation are,

$$s = \frac{-2\omega\delta_n \pm \sqrt{(2\delta\omega_n)^2 - 4\omega_n^2}}{2} = \frac{-2(\delta\omega_n \pm \omega_n\sqrt{\delta^2-1})}{2}$$

$$\Rightarrow s = -\delta\omega_n \pm \omega_n\sqrt{\delta^2-1}$$

When = 0, the two roots are fictitious,

When = 1, the two roots were real and equal.

Thetworootsarerealbutnotequalwhenδ >1.

Thetworootsarecomplexconjugatewhen0 <δ<1.Wecanwrite C(s)equationas,

$$C(s) = \left(\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}\right)R(s)$$

Where,

The output signal's Laplace transform is represented by C(s) (t)

The input signal's Laplace transform is represented by R(s), the natural frequency by r(t), and the damping ratio by r(s).

Get the response (output) of a second order system inside the time domain by following these procedures.

Take Laplace transform of the input signal, r (t)

Consider the equation,

$$\left(\frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}\right) R(s)$$

Substitute R(s) value in the above equation.

Do partial fractions of C'(s) if required.

Apply inverse Laplace transform to C'(s).

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CHAPTER 9

SECOND-ORDER STEP RESPONSE SYSTEM

Dr. Govind Singh, Assistant Professor
Department of Computer Science and Engineering, Sanskriti University, Mathura, Uttar Pradesh, India
Email Id- govind@sanskriti.edu.in

Second-Order Step Response System

Think of the second order system's input as comprising the unit step signal. The unit step signal's Laplace transform is,

R(s) = 1/

We know the transfer function of the second order closed loop control system is,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

Case 1: ō = 0

Substitute, & = 0 in the transfer function,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{s^2 + \omega_n^2}\right) R(s)$$

Substitute, R(s) =1/s in the above equation,

$$C(s) = \left(\frac{\omega_n^2}{s^2 + \omega_n^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)}$$

Apply inverse Laplace transform on both the sides,

$$c(t) = (1 - \cos(\omega_n t))\, u(t)$$

So, the unit step response of the second order system when /delta = 0 will be a continuous time signal with constant amplitude and frequency.

Case 2: ō = 1

Substitute, /delta = 1 in the transfer function,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s + \omega_n)^2}\right) R(s)$$

Substitute, R(s) =1/s in the above equation,

$$C(s) = \left(\frac{\omega_n^2}{(s+\omega_n)^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s+\omega_n)^2}$$

Do partial fractions of C'(s),

$$C(s) = \frac{\omega_n^2}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

After simplifying, you will get the values of A, B and C as 1, -1 and respectively. Substitute these values in the above partial fraction expansion of C'(s),

$$c(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})u(t)$$

Apply inverse Laplace transform on both the sides,

$$c(t) = (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})u(t)$$

So, the unit step response of the second order system will try to reach the step input in steady state.

Case 3: 0 < 5 < 1

We can modify the denominator term of the transfer function as follows,

$$s^2 + 2\delta\omega_n s + \omega_n^2 = \left\{s^2 + 2(s)(\delta\omega_n) + (\delta\omega_n)^2\right\} + \omega_n^2 - (\delta\omega_n)^2$$
$$= (s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)$$

The transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}\right) R(s)$$

Substitute, R(s) =1/s in the above equation,

$$C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s\left((s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)\right)}$$

Do partial fractions of C(s),

$$C(s) = \frac{\omega_n^2}{s\left((s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)\right)} = \frac{A}{s} + \frac{Bs+C}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

After simplifying, you will get the values of A, B and C as 1, -1 and - 28wn respectively. Substitute these values in the above partial fraction expansion of C(s),

$$C(s) = \frac{1}{s} - \frac{s+2\delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

$$C(s) = \frac{1}{s} - \frac{s+\delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)} - \frac{\delta\omega_n}{(s+\delta\omega_n)^2 + \omega_n^2(1-\delta^2)}$$

$$C(s) = \frac{1}{s} - \frac{(s+\delta\omega_n)}{(s+\delta\omega_n)^2 + (\omega_n\sqrt{1-\delta^2})^2} - \frac{\delta}{\sqrt{1-\delta^2}}\left(\frac{\omega_n\sqrt{1-\delta^2}}{(s+\delta\omega_n)^2 + (\omega_n\sqrt{1-\delta^2})^2}\right)$$

Substitute, $\omega_n\sqrt{1-\delta^2}$ as ω_d in the above equation

$$C(s) = \frac{1}{s} - \frac{(s+\delta\omega_n)}{(s+\delta\omega_n)^2 + \omega_d^2} - \frac{\delta}{\sqrt{1-\delta^2}}\left(\frac{\omega_d}{(s+\delta\omega_n)^2 + \omega_d^2}\right)$$

Apply inverse Laplace transform on both the sides,

$$c(t) = \left(1 - e^{-\delta\omega_n t}\cos(\omega_d t) - \frac{\delta}{\sqrt{1-\delta^2}}e^{-\delta\omega_n t}\sin(\omega_d t)\right)u(t)$$

$$c(t) = \left(1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\left((\sqrt{1-\delta^2})\cos(\omega_d t) + \delta\sin(\omega_d t)\right)\right)u(t)$$

-If √1 – 62 = sin (), then'd' will be cos (8). Substitute these values in the above equation,

$$c(t) = \left(1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}(\sin(\theta)\cos(\omega_d t) + \cos(\theta)\sin(\omega_d t))\right)u(t)$$

$$\Rightarrow c(t) = \left(1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)\right)u(t)$$

Therefore, when is between zero and one, the unit step response of the second order system exhibits damped oscillations (decreasing amplitude)[1]–[6].

Case4:δ> 1

The transfer function's denominator term may be changed as follows

$$s^2 + 2\delta\omega_n s + \omega_n^2 = \{s^2 + 2(s)(\delta\omega_n) + (\delta\omega_n)^2\} + \omega_n^2 - (\delta\omega_n)^2$$

$$= (s + \delta\omega_n)^2 - \omega_n^2(\delta^2 - 1)$$

The transfer function becomes,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{(s+\delta\omega_n)^2 - \omega_n^2(\delta^2 - 1)}$$

$$\Rightarrow C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 - \omega_n^2(\delta^2 - 1)}\right) R(s)$$

Substitute, R(s) = 1/s in the above equation,

$$C(s) = \left(\frac{\omega_n^2}{(s+\delta\omega_n)^2 - (\omega_n\sqrt{\delta^2-1})^2}\right)\left(\frac{1}{s}\right) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

C'(s) = Do partial fractions of C'(s),

$$C(s) = \frac{\omega_n^2}{s(s+\delta\omega_n+\omega_n\sqrt{\delta^2-1})(s+\delta\omega_n-\omega_n\sqrt{\delta^2-1})}$$

$$= \frac{A}{s} + \frac{B}{s+\delta\omega_n+\omega_n\sqrt{\delta^2-1}} + \frac{C}{s+\delta\omega_n-\omega_n\sqrt{\delta^2-1}}$$

After simplifying, you will get the values of A, B and C as 1, 1.

$$\frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}$$

and respectively. Substitute these values in above partial fraction expansion of C'(s),

$$C(s) = \frac{1}{s} + \frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\left(\frac{1}{s+\delta\omega_n+\omega_n\sqrt{\delta^2-1}}\right)$$
$$-\left(\frac{1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)\left(\frac{1}{s+\delta\omega_n-\omega_n\sqrt{\delta^2-1}}\right)$$

Apply inverse Laplace transform on both the sides,

$$c(t)$$
$$= \left(1 + \left(\frac{1}{2(\delta+\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)e^{-(\delta\omega_n+\omega_n\sqrt{\delta^2-1})t}\right.$$
$$\left. - \left(\frac{1}{2(\delta-\sqrt{\delta^2-1})(\sqrt{\delta^2-1})}\right)e^{-(\delta\omega_n-\omega_n\sqrt{\delta^2-1})t}\right)u(t)$$

Due to overdamping, the second order system's unit step response when > 1 will never exceed step input in the steady state.

Second-order r system impulse response

Any one of these two approaches may be used to determine the impulse response of the second order system.

Follow the steps involved in deriving step response by assuming R(s) to have a value of 1 rather than 1/s.

Make the step response distinction.The following table displays the second order system's impulse response for four different damping ratio situations.

Condition of Damping ratio	Impulse response for t ≥ 0
$\delta = 0$	$\omega_n \sin(\omega_n t)$
$\delta = 1$	$\omega_n^2 t e^{-\omega_n t}$
$0 < \delta < 1$	$\left(\frac{\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t)$
$\delta > 1$	$\left(\frac{\omega_n}{2\sqrt{\delta^2-1}}\right)\left(e^{-(\delta\omega_n-\omega_n\sqrt{\delta^2-1})t} - e^{-(\delta\omega_n+\omega_n\sqrt{\delta^2-1})t}\right)$

Let's discuss about the second order system's time domain requirements in this chapter. The following graphic displays the step response of the second order system for the underdamped situation. This diagram depicts every time domain definition[7]–[10]. The term "transient response" refers to the reaction before to the settling period, while the term "steady state response" refers to the response after the settling time in Figure 9.1.

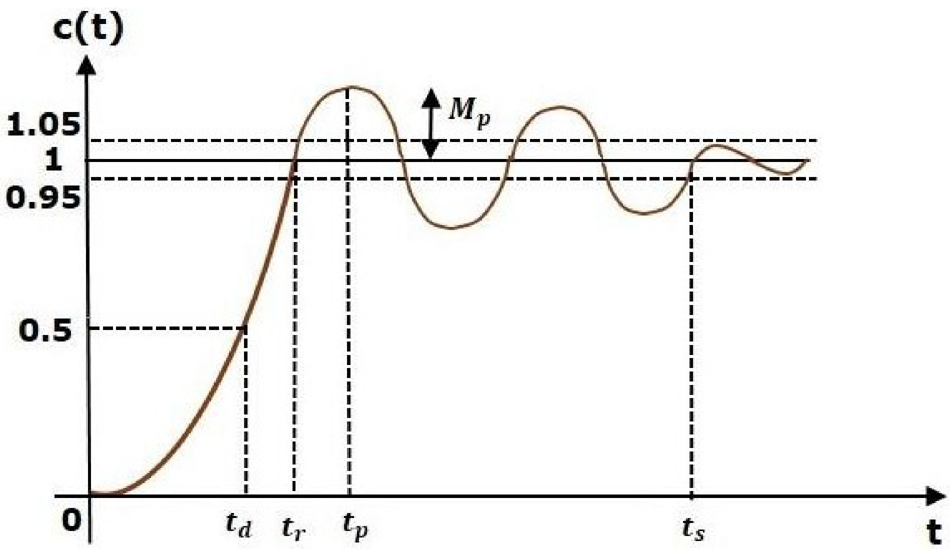

Figure 9.1: diagram depicts every time domain.

Delay Period

It is the amount of time needed from the zero instant for the reaction to reach half of its ultimate value. The symbol for it is tdtd. When " is between zero and one, take into consideration the step response of the second order system for t 0.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

The final value of the step response is one.

Therefore, at t=ta, the value of the step response will be 0.5. Substitute, these values in the above equation.

$$c(t_d) = 0.5 = 1 - \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_d + \theta) = 0.5$$

By using linear approximation, you will get the delay time to as,

$$t_d = \frac{1 + 0.7\delta}{\omega_n}$$

RiseTime

It measures how long it takes a reaction to increase from 0% to 100% of its ultimate value. This is true for underdamped systems. Consider the duration from 10% to 90% of the final value again for overdamped systems. The symbol for rise time is tr.

c(t) = 0 at time t = t1 = 0.

We are aware that the step response's ultimate value is one. As a result, step response has a value of 1 at time t=t2. Replace these values in the equation below.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

$$c(t_2) = 1 = 1 - \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \omega_d t_2 + \theta = \pi$$

$$\Rightarrow t_2 = \frac{\pi - \theta}{\omega_d}$$

Substitute t□ and to values in the following equation of rise time,

$$t_r = t_2 - t_1$$

$$\therefore\ t_r = \frac{\pi - \theta}{\omega_d}$$

I may infer from the preceding equation that the relationship between the rising time (tr) and the damped frequency (d) is inverse.

Peak Period

It is the amount of time needed for the reaction to first reach its highest value. The symbol for it is tp. The response's initial derivate is zero at t=tp.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Differentiate $c(t)$ with respect to 't'.

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = -\left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\omega_d \cos(\omega_d t + \theta) - \left(\frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

They are aware that the second order system's step response in the underdamped scenario is.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Differentiate $c(t)$ with respect to 't'.

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = -\left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\omega_d \cos(\omega_d t + \theta) - \left(\frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Substitute, $t = t_p$ and $\frac{\mathrm{d}c(t)}{\mathrm{d}t} = 0$ in the above equation.

$$0 = -\left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right)\left[\omega_d \cos(\omega_d t_p + \theta) - \delta\omega_n \sin(\omega_d t_p + \theta)\right]$$

$$\Rightarrow \omega_n\sqrt{1-\delta^2}\cos(\omega_d t_p + \theta) - \delta\omega_n \sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sqrt{1-\delta^2}\cos(\omega_d t_p + \theta) - \delta \sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\theta)\cos(\omega_d t_p + \theta) - \cos(\theta)\sin(\omega_d t_p + \theta) = 0$$

$$\Rightarrow \sin(\theta - \omega_d t_p - \theta) = 0$$

$$\Rightarrow sin(-\omega_d t_p) = 0 \Rightarrow -\sin(\omega_d t_p) = 0 \Rightarrow sin(\omega_d t_p) = 0$$

$$\Rightarrow \omega_d t_p = \pi$$

Differentiate c(t) with respect to 't',

$$\frac{dc(t)}{dt} = -\left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\omega_d \cos(\omega_d t + \theta) - \left(\frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

Substitute, ttp and de(t) dt 0 in the above equation,

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t + \theta)$$

$$\Rightarrow t_p = \frac{\pi}{\omega_d}$$

One may infer from the aforementioned equation that the relationship between the peak time tp and the damped frequency d is inverse.

Maximum Overshoot

Maximum overshoot Mp is described as the difference between the reaction at its peak and the response's overall value. Additionally known as the greatest overshoot.

It may be expressed mathematically as Mp=c(tp) c()

Where c(tp) denotes the response's peak value and c() denotes the response's ultimate (steady state) value.

The result of c(t) at time t=tp is.

$$c(t_p) = 1 - \left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right)\sin(\omega_d t_p + \theta)$$

Substitute, $t_p = \frac{\pi}{\omega_d}$ in the right hand side of the above equation.

$$c(t_P) = 1 - \left(\frac{e^{-\delta\omega_n\left(\frac{\pi}{\omega_d}\right)}}{\sqrt{1-\delta^2}}\right)\sin\left(\omega_d\left(\frac{\pi}{\omega_d}\right) + \theta\right)$$

$$\Rightarrow c(t_p) = 1 - \left(\frac{e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}}{\sqrt{1-\delta^2}}\right)(-\sin(\theta))$$

We know that,

$$\sin(\theta) = \sqrt{1-\delta^2}$$

o, we will get c(tp) as,

$$c(t_p) = 1 + e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}$$

Substitute the values of c(tp) and c(∞) in the peak overshoot equation,

$$M_p = 1 + e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)} - 1$$

$$\Rightarrow M_p = e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}$$

Percentage of peak overshoot % Mp can be calculated by using this formula.

$$\%M_p = \frac{M_p}{c(\infty)} \times 100\%$$

From Using the equation above, can can deduce that when the damping ratio rises, the percentage of peak overshoot%Mp will decrease.

Resolving period

It is the amount of time necessary for the reaction to stabilize and remain within the designated tolerance ranges around the final result. The tolerance bands typically range between 2% and 5%. Ts is used to indicate the settling time.

Thesettlingtimefor5%tolerancebandis.

$$t_s = \frac{3}{\delta\omega_n} = 3\tau$$

Thesettlingtimefor2%tolerancebandis.

$$t_s = \frac{4}{\delta\omega_n} = 4\tau$$

Where 1/n is the value of the time constant,

The damping ratio has an inverse relationship with the settling time ts and the time constant.

The system gain has no bearing on the settling time ts or the time constant. This implies that even if the system gain changes, the settling time ts and time constant won't.

Example

When the unit step signal is supplied as an input to this control system, let's discover the time domain requirements of the control system with the closed loop transfer function.

They are aware of the second order closed loop control system's typical form of the transfer function as.

$$\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

One may calculate the un-damped natural frequency n as 2 rad/sec and the damping ratio as 0.5 by equating these equal transfer functions.The equation for damped frequency d is known as.

$$\omega_d = \omega_n \sqrt{1-\delta^2}$$

$$\omega_d = \omega_n \sqrt{1-\delta^2}$$

Substitute, ω_n and δ values in the above formula.

$$\Rightarrow \omega_d = 2\sqrt{1-(0.5)^2}$$

$$\Rightarrow \omega_d = 1.732\, rad/sec$$

Substitute, δ value in following relation

$$\theta = \cos^{-1}\delta$$

$$\Rightarrow \theta = \cos^{-1}(0.5) = \frac{\pi}{3}\, rad$$

To get the values for time domain specifications for a particular transfer function, substitute the aforementioned requisite values into each time domain specification's formula and simplify.The formulas for time domain requirements, required value substitutions, and final values are shown in the following table.

Time domain specification	Formula	Substitution of values in Formula	Final value
Delay time	$t_d = \frac{1+0.7\delta}{\omega_n}$	$t_d = \frac{1+0.7(0.5)}{2}$	t_d =0.675 sec
Rise time	$t_r = \frac{\pi-\theta}{\omega_d}$	$t_r = \frac{\pi-(\frac{\pi}{3})}{1.732}$	t_r =1.207 sec
Peak time	$t_p = \frac{\pi}{\omega_d}$	$t_p = \frac{\pi}{1.732}$	t_p =1.813 sec
% Peak overshoot	$\%M_p = \left(e^{-\left(\frac{\delta\pi}{\sqrt{1-\delta^2}}\right)}\right) \times 100\%$	$\%M_p = \left(e^{-\left(\frac{0.5\pi}{\sqrt{1-(0.5)^2}}\right)}\right) \times 100\%$	$\% M_p$ =16.32%
Settling time for 2% tolerance band	$t_s = \frac{4}{\delta\omega_n}$	$t_S = \frac{4}{(0.5)(2)}$	t_s =4 sec

Stable state error is the difference between the output of the control system and the expected response at steady state. The symbol for it is ess. The following is how the final value theorem may be used to determine steady state error.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} E(s)$$

Where E(s) is the error signal's Laplace transform, and e (t)

Let's go through how to determine steady state faults for control systems with unity feedback and without it one at a time.

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CHAPTER 10

STEADY STATE ERRORS

Dr. Arvind Kumar Pal, Associate Professor
Department of Computer Science and Engineering, Sanskriti University, Mathura, Uttar Pradesh, India
Email Id- arvind@sanskriti.edu.in

Steady State Errors in Feedback Systems in Unity

Take a look at the closed loop control system block diagram below, which has a single negative feedback signal (Figure 9.2).

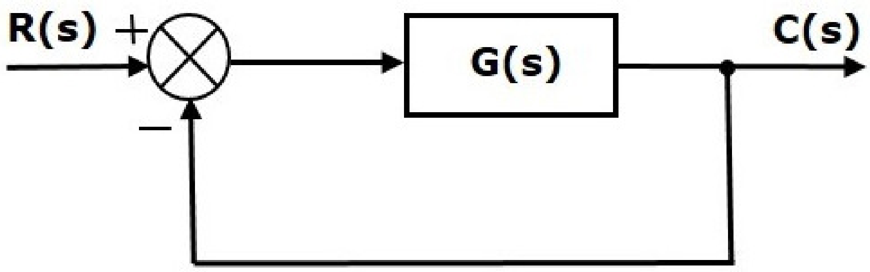

Figure 9.2 Steady State Errors in Feedback Systems in Unity

Where,

= R(s) is the Laplace transform of the reference Input signal r(t)

=C(s) is the Laplace transform of the output signal c(t)

We know the transfer function of the unity negative feedback closed loop control system as,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1+G(s)}$$

The output of the summing point is,

$$E(s) = R(s) - C(s)$$

Substitute C(s) value in the above equation,

$$E(s) = R(s) - \frac{R(s)G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s) + R(s)G(s) - R(s)G(s)}{1+G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1+G(s)}$$

Substitute E(s) value in the steady state error formula,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)}$$

For common input signals including unit step, unit ramp, and unit parabolic signals, the following table displays steady state errors and error constants.

Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p = \lim_{s \to 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v = \lim_{s \to 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a = \lim_{s \to 0} s^2G(s)$

Where Kp, Kv, and Ka stand for the relevant position, velocity, and acceleration error constants.

Note: Multiply the relevant steady state error by the amplitude of any of the aforementioned input signals whose amplitude is different from unity[1]–[6].

Note – since it only occurs at the origin, we are unable to determine the steady state error for the unit impulse signal. Therefore, because t signifies infinity, we are unable to compare the impulse response with both the unit impulse input.

Example

(5+ Let us find the steady state error for an input signal r (t) = 5+2t+ = 5(s+4) s^2(s+1) (s+20) of unity negative feedback control system with G(s): = 2) u(t)

The given input signal is a combination of three signals step, ramp and parabolic. The following table shows the error constants and steady state error values for these three signals.

Input signal	Error constant	Steady state error
$r_1(t) = 5u(t)$	$K_p = \lim_{s\to 0} G(s) = \infty$	$e_{ss1} = \frac{5}{1+k_p} = 0$
$r_2(t) = 2tu(t)$	$K_v = \lim_{s\to 0} sG(s)$ $= \infty$	$e_{ss2} = \frac{2}{K_v} = 0$
$r_3(t) = \frac{t^2}{2}u(t)$	$K_a = \lim_{s\to 0} s^2G(s)$ $= 1$	$e_{ss3} = \frac{1}{k_a} = 1$

Wewillgettheoverallsteadystateerror,byaddingtheabovethreesteadystateerrors.

ess=ess1+ess2+ess3

⇒ess=0+0+1=1⇒ess=0+0+1=1

Therefore,wegotthesteadystateerroressas**1** forthisexample.

SteadyStateErrors forNon-UnityFeedbackSystems

Considerthefollowingblockdiagramofclosedloopcontrolsystem,whichishavingnonunitynegativ efeedback in Figure 9.3.

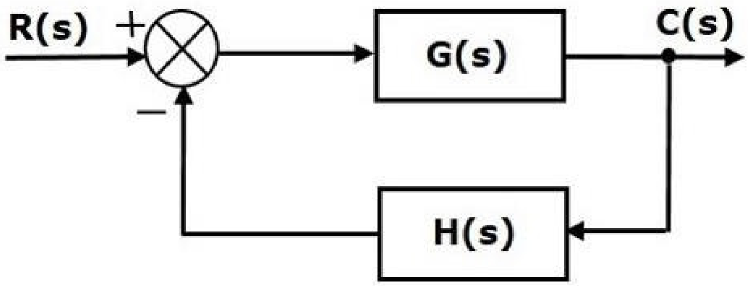

Figure 9.3: Illustrates the Blockdiagramofclosedloopcontrolsystem.

Only for feedback systems with unity, can we calculate steady state errors. Therefore, the non-unity feedback system has to be changed to the unity feedback system. In the block diagram above, add one unity positive feedback line and one unity negative feedback path to account for this. The updated block diagram appears as follows in Figure 9.4.

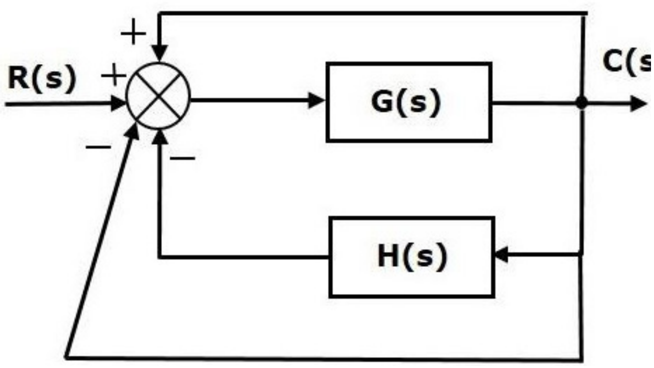

Figure 9.4: Illustrates the updated block diagram closed loop control system.

By leaving the unity negative feedback in its current state, simplify the block diagram above. This is the abbreviated block diagram in Figure 10.3.

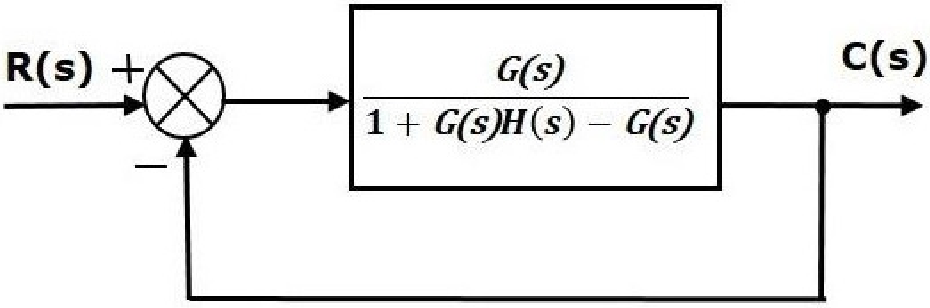

Figure 9.5: Illustrates the leaving the unity negative feedback in its current state.

The block diagram of the unity negative feedback closed loop control system is similar to this one. In this case, the single block's transfer function is G(s) / [1+G(s)H(s)G(s)] rather than G.(s). The steady state error formula for unity negative feedback systems may now be used to compute the steady state errors[7]–[12].

Note: For unstable closed loop systems, determining the steady state errors is useless. Therefore, only closed loop stable systems must be considered for calculating steady state errors. This implies that before identifying the steady state faults, we must determine if the control system is stable. We will go through stability-related ideas in the next chapter.

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CHAPTER 11

STABILITY ANALYSIS IN S-DOMAIN

Dr. Deepanshu Singh, Assistant Professor
Department of Computer Science and Engineering, Sanskriti University, Mathura, Uttar Pradesh, India
Email Id- deepanshu@sanskriti.edu.in

StabilityAnalysisinS-Domain

It's crucial to understand stability. Let's talk about system stability and system kinds based on stability in this chapter.

Stability

If a system's output is under control, it is considered to be stable. It is believed to be unstable if not. For a given bounded input, a stable system generates a bounded output. The reaction of a stable system is shown in the next picture in Figure 10.1.

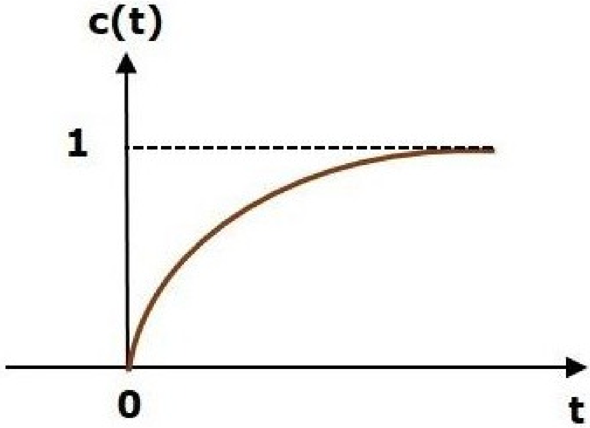

Figure 10.1: Illustrates the reaction of a stable system.

This is the first order control system's reaction to the unit step input. This response's values range from 0 to 1. It is hence bounded output. We are aware that for all positive values of t, including zero, the unit step signal has a value of 1. Input is thus bounded. Since both the input and the output are bounded, the first order control system is stable[1]–[3].

Systems based on Types of Stability

1. According to their stability, the systems may be categorized as follows.
2. Unquestionably stable system
3. System that is only stably stable
4. Possibly unstable system
5. System that is utterly stable

The system is referred to as being perfectly stable if it remains stable over the whole range of system component values. If all of the open loop transfer function's poles are located on the left half of the's' plane, the open loop control system is perfectly stable. Similarly, if all of the closed loop transfer function's poles are located in the left side of the's' plane, the closed loop control system is completely stable.

Constrained Stability System

A system is said to be conditionally stable if it is stable throughout a range of values for each system component.

System with Marginal Stability

A system is considered moderately stable if it produces an output signal with constant amplitude and constant frequency of oscillations given limited input. If any two poles of a open loop transfer function are present on the hypothetical axis, the open loop control system is only partially stable. Similar to this, if any two poles of a closed loop transfer function are located on the imaginary axis, the closed loop control system is only partially stable. Let's talk about the Routh-Hurwitz stability criterion-based stability analysis in the's' domain in this chapter. In order to determine the stability of closed loop control systems, we need the characteristic equation in this criteria.

Routh-Hurwitz Stability Criterion

Having one required requirement and one sufficient condition for stability is the Routh-Hurwitz stability criteria. Any control system that doesn't fulfill the prerequisite requirement is unstable, according to our definition. However, if the required condition is met, the control system may or may not be stable. In order to determine if the control system is stable or not, the adequate condition is useful.

Condition required for Routh-Hurwitz Stability

The characteristic polynomial's coefficients must be positive in order for the condition to exist. This indicates that there should be negative real portions in all of the characteristic equation's roots.

Considerthe characteristicequationofthe order‘n’is-

$$a_0s^n + a_1s^{n-1} + a_2s^{n-2} + \ldots + a_{n-1}s^1 + a_ns^0 = 0$$

Notably, the nth order characteristic equation shouldn't include any missing terms. This implies that there shouldn't be any zero-valued coefficients in the nth order characteristic equation.

Condition sufficient for Routh-Hurwitz Stability

The Routh array's first column should include only entries with the same sign, and this is the necessary requirement. This implies that all of the entries in the Routh array's first column must be either positive or negative.

Array Routh Method

The control system is stable if all of the roots of a characteristic equation can be found on the left half of the's' plane. The control system is unstable if at least one characteristic equation root is located in the right-half of the "s" plane. To determine the weather control system is steady or unstable, we must locate the characteristic equation's roots. However, as order rises, it becomes more difficult to identify the characteristic equation's roots. We thus have the Routh array technique to solve this issue. The characteristic equation's roots do not need to be calculated using this approach. Create the Routh table first, then look up the number of sign changes in the first column. The number of sign shifts in the first column of a Routh table indicates the number of characteristic equation roots present in the right half of the's' plane and the instability of the control system.

To construct the Routh table, follow these steps.

The coefficients of a characteristic polynomial as shown in the table below should fill the first two rows of the Routh array. Up to the coefficient of s0, start with the sn coefficient.

The items listed in the table below should be inserted into the remaining rows of the Routh array. Keep going until you reach the first column element of row s0s0, which is an. In this case, a represents the characteristic polynomial's s0 coefficient.

Note: If any of the row components in the Routh table have a common factor, you may divide the row elements by that factor to simplify things. The Routh array of the nth order characteristics polynomial is shown in the table below.

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s^1 + a_n s^0$$

s^n	a_0	a_2	a_4	a_6	$\ldots$	$\ldots$
s^{n-1}	a_1	a_3	a_5	a_7	$\ldots$	$\ldots$
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$	$b_3 = \frac{a_1 a_6 - a_7 a_0}{a_1}$	$\ldots$	$\ldots$	$\ldots$
s^{n-3}	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	$c_2 = \frac{b_1 a_5 5 - b_3 a_1}{b_1}$	$\vdots$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
s^1	$\vdots$	$\vdots$				
s^0	a_n					

Example:

Letusfindthestabilityofthecontrolsystemhavingcharacteristicequation,

$$s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$$

Step1–Verifythe necessaryconditionforthe Routh-Hurwitzstability.Allthecoefficients ofthecharacteristicpolynomial, arepositive.So,thecontrolsystemsatisfiesthenecessary condition.

Step2–FormtheRoutharrayforthe givencharacteristicpolynomial.

$s^4 + 3s^3 + 3s^2 + 2s + 1$

s^4	1	3	1
s^3	3	2	
s^2	$\frac{(3\times3)-(2\times1)}{3} = \frac{7}{3}$	$\frac{(3\times1)-(0\times1)}{3} = \frac{3}{3} = 1$	
s^1	$\frac{\left(\frac{7}{3}\times2\right)-(1\times3)}{\frac{7}{3}} = \frac{5}{7}$		
s^0	1		

Step 3: Confirm the Routh-Hurwitz stability's necessary conditions.

The Routh array's first column contains only positive items. In the first column of the Routh array, there is no change in sign. The control mechanism is hence stable.

Individual Routh Array Cases

While constructing the Routh table, we could encounter two different kinds of scenarios. It is challenging to finish the Routh table from these two examples.

These are the two exceptional cases:

In the Routh's array, zero is the initial entry in any row.

The Routh's array has zero items in every row.

Let's now go through each of these two situations' challenges individually.

The Routh's array's first element in any row is zero[4]–[13].

If any row in the Routh's array only has the first element set to zero and at least one other element does have a value other than zero, the first element should be changed to a tiny positive integer,, in that row. Afterward, go on with completing Routh's table. By replacing tends to zero, determine the number of sign changes within the first column of Routh's table.

Example

$$s^4 + 2s^3 + s^2 + 2s + 1 = 0$$

Letusfindthestabilityofthecontrolsystemhavingcharacteristicequation,

Step1–Verifythe necessaryconditionforthe Routh-Hurwitzstability.Allthecoefficients ofthecharacteristicpolynomial, are optimistic. As a result, the control system met the prerequisite.

Form the Routh array for the provided characteristic polynomial in step two.

$$s^4 + 2s^3 + s^2 + 2s + 1 = 0$$

s^4	1	1	1
s^3	~~2~~ 1	~~2~~ 1	
s^2	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(0\times1)}{1} = 1$	
s^1			
s^0			

The common factor for the s3 items in the row is 2. So, we split each of these components by 2. Only the first element in row s2 is zero in special instance I Therefore, change it to and continue completing this same Routh table by replacing it with.

s^4	1	1	1
s^3	1	1	
s^2	ϵ	1	
s^1	$\frac{(\epsilon\times1)-(1\times1)}{\epsilon} = \frac{\epsilon-1}{\epsilon}$		
s^0	1		

Step3–Verifythe sufficientconditionforthe Routh-Hurwitzstability.

As□tendstozero,theRouthtablebecomeslikethis.

s^4	1	1	1
s^3	1	1	
s^2	0	1	
s^1	-∞		
s^0	1		

The first column of the Routh table has two sign changes. The control system is thus unsteady.

The Routh's array has zero items in every row.

Take these two actions in this situation. –

The row right above the row of zeros has an auxilary equation, A(s), which should be written down.

the difference between the auxiliary equation A(s) and s. With these coefficients, complete the zeros in the row.

Example

$$s^5+3s^4+s^3+3s^2+s+3=0$$

Let's determine if a control system with a characteristic equation is stable,

Step 1: Confirm the Routh-Hurwitz stability's preconditions.

The provided characteristic polynomial has only positive coefficients. As a result, the control system met the prerequisite. Form the Routh array for the provided characteristic polynomial in step two.

s^5	1	1	1
s^4	~~3~~ 1	~~3~~ 1	~~3~~ 1
s^3	$\frac{(1\times1)-(1\times1)}{1}=0$	$\frac{(1\times1)-(1\times1)}{1}=0$	
s^2			
s^1			
s^0			

The row s^1 elements have the common factor of 3. So, all these elements are divided by 3.

Special case (ii) - All the elements of row s^3 are zero. So, write the auxiliary equation, A(s) of the row s^1.

$$A(s) = s^4 + s^2 + 1$$

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

Place these coefficients in row s^3,

s^5	1	1	1
s^4	1	1	1
s^3	~~4~~ 2	~~2~~ 1	
s^2	$\frac{(2\times1)-(1\times1)}{2} = 0.5$	$\frac{(2\times1)-(0\times1)}{2} = 1$	
s^1	$\frac{(0.5\times1)-(1\times2)}{0.5} = \frac{-1.5}{0.5}$ $= -3$		
s^0	1		

Step 3: Confirm the Routh-Hurwitz stability's necessary conditions.

The first column of the Routh table has two sign changes. The control system is thus unsteady.

We may determine if the closed loop poles are in the left half of the's' plane, the right half of the's' plane, or on a hypothetical axis using the Routh-Hurwitz stability criteria. Thus, we are unable to determine the kind of control system. The root locus method may be used to get around this restriction.

Root-locus Method

We can see the closed loop poles' route in the root locus graphic. As a result, we can determine the kind of control system. In order to determine the stability of the closed loop control system, we shall employ an open loop transfer function in this method.

Principles of Root Locus

The Root locus, which may be changed by altering system gain K from zero to infinity, is the location of the roots of the characteristic equation.

Weknowthat,thecharacteristicequationoftheclosedloopcontrolsystemis,

$$1 + G(s)H(s) = 0$$

We can represent G(s)H(s) as,

$$G(s)H(s) = K\frac{N(s)}{D(s)}$$

Where,

K represents the multiplying factor

N(s) represents the numerator term having (factored) nth order polynomial of 's'.

D(s) represents the denominator term having (factored) mth order polynomial of 's'.

Substitute, G(s)H(s) value in the characteristic equation,

$$1 + k\frac{N(s)}{D(s)} = 0$$

$$\Rightarrow D(s) + KN(s) = 0$$

Case 1 - K=0

If K = 0, then D(s) = 0.

That means, the closed loop poles are equal to open loop poles when K is zero.

Case 2 - K = ∞

Re-write the above characteristic equation as,

$$K\left(\frac{1}{K} + \frac{N(s)}{D(s)}\right) = 0 \Rightarrow \frac{1}{K} + \frac{N(s)}{D(s)} = 0$$

Substitute, K = ∞ in the above equation,

$$\frac{1}{\infty} + \frac{N(s)}{D(s)} = 0 \Rightarrow \frac{N(s)}{D(s)} = 0 \Rightarrow N(s) = 0$$

If K = ∞, then N(s) = 0. It means the closed loop poles are equal to the open loop zeros when K is infinity.

Fromabovetwocases,wecanconcludethattherootlocusbranchesstartatopenlooppolesandendatop enloopzeros.

AngleConditionandMagnitudeCondition

The angles are met by the points on the root locus branches. In order to determine if a point exists on the root locus branch or not, the angle condition is utilized. Utilizing the magnitude condition, we can determine the value of K for the locations on the root locus branches. Since the angle requirement is satisfied, we can employ the magnitude condition for the points.

The following equation represents a closed loop control system.

$$1 + G(s)H(s) = 0$$

$$\Rightarrow G(s)H(s) = -1 + j0$$

The **phase angle** of $G(s)H(s)$ is

$$\angle G(s)H(s) = \tan^{-1}\left(\frac{0}{-1}\right) = (2n+1)\pi$$

The **angle condition** is the point at which the angle of the open loop transfer function is anoddmultipleof180^0.

MagnitudeofG(s),H(s)G(s)H(s)is.

$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2} = 1$$

The point at which the angle requirement is met and the magnitude of the open loop transfers function is just one is known as the magnitude condition. The root locus is shown graphically in the s-domain and has symmetrical properties around the real axis. Due to the fact that the open loop poles and zeroes are present in the s-domain and have values that might be either real or complex conjugate pairs. Let's talk about how to create (draw) the root locus in this chapter.

Rules for Building a Root Locus

Use these guidelines while creating a root locus.

Rule 1: In the s' plane, find the open loop poles and zeros.

Rule 2: Count the branches at the root locus.

We are aware that the open loop zeros and poles mark the beginning and end of the root locus branches. In other words, the number of finite open loop poles P or zeros Z, whichever is bigger, equals the number of root locus branches N.

The number of root locus branches N may be expressed mathematically as N=P if PZ N=Z if PZ.

Rule 3: Locate and sketch the true axis root and locus branches.

A point is on the root locus if the angle of a open loop transfer function there is an odd multiple of 1800. A point here on real axis is on the root locus branch if an odd number of a open loop poles plus zeros are present to its left. The branch of points that meets this requirement is therefore the true axis of a root locus branch.

Rule 4: Determine the asymptote's centroid and angle.

The root locus branches all begin at finite open loop poles and finish at finite open loop zeros if P=Z.

In the event where P>Z, then PZ number of root locus branches begin at finite open loop poles and finish at infinite open loop zeros, as opposed to Z number of root locus branches beginning at finite open loop poles and ending at finite open loop zeros.

In the event where P > Z, P number of root locus branches begin at finite open loop poles and finish at finite open loop zeros, while Z number of root locus branches begin at infinite open loop poles and conclude at finite open loop zeros.

Therefore, when P > Z, some of the root locus branches reach infinity. The orientation of these root locus branching is provided by asymptotes. Centroid is the location where asymptotes cross the real axis.

Wecancalculatethe**centroidα** byusingthisformula,

$$\alpha = \frac{\sum Real\ part\ of\ finite\ open\ loop\ poles - \sum Real\ part\ of\ finite\ open\ loop\ zeros}{P-Z}$$

The formula for the angle of **asymptotes θ** is

$$\theta = \frac{(2q+1)180^0}{P-Z}$$

Where,

$$q = 0, 1, 2, \ldots, (P-Z)-1$$

Rule 5: Locate the spots where the branches of the root locus cross a hypothetical axis. Using the Routh array approach and a specific case, we can determine the position where the root locus branch crosses the imaginary axis and the value of K at that location (ii).

The root locus branch contacts the imaginary axis if all elements in any row of the Routh array is zero, and vice versa. Determine the row such that if we set the first element to zero, the items in the whole row will also be set to zero. Determine K's value for this combo. Put this K value in the auxiliary equation as a replacement. You will discover where the root locus branch crosses a hypothetical axis.

Find break-away and break-in points according to Rule 6.

There will be a break-away point between these three open loop poles if there is a true axis root locus branch between them. A break-in point will be present between two open loop zeros when there is a real axis root locus branch present between them. Note: Only the true axis root locus branches have break-away and break-in locations. To locate break-away and break-in spots, follow these instructions.

From the characteristic equation 1+G(s)H(s)=0, write K in terms of s. K should be differentiated with respect to s then set to zero. Replace these ss numbers in the equation above. The break points are ss values in which the K value is positive. Rule 7: Determine the angles of departure and arrival. At the complex conjugate open loop poles and zeros, respectively, the angle of departure as well as the angle of arrival may be determined. The angle of departure (d) formula is,

$$\phi_d = 180^0 - \phi$$

The formula for the **angle of arrival** ϕ_a is

$$\phi_a = 180^0 + \phi$$

Where,

$$\phi = \sum \phi_P - \sum \phi_Z$$

Example

Letusnowdrawtherootlocusofthecontrolsystemhavingopenlooptransfer function,

$$G(s)H(s) = \frac{K}{s(s+1)(s+5)}$$

Step1−thegivenopenloop transferfunctionhasthreepoles ats=0,

s=-1,s=-5.Itdoesn'thaveanyzero.Therefore,thenumberofrootlocusbranchesisequaltothenumberofpoles oftheopenloop transferfunction in Figure 10.2. N=P=3

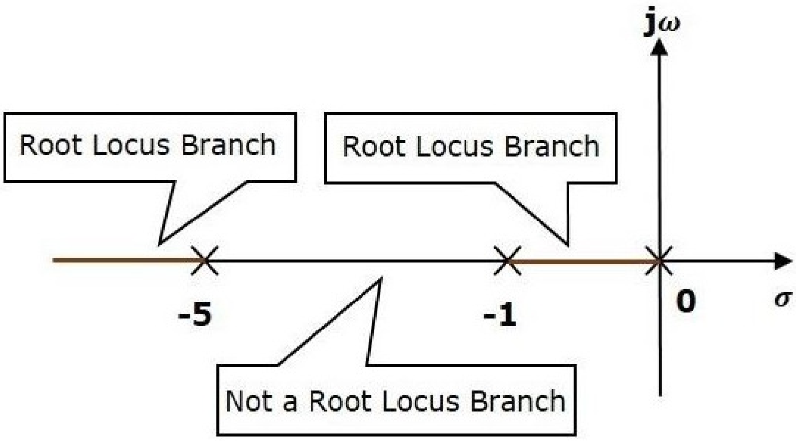

Figure 10.2: Illustrates thenumberofrootlocusbranchesisequaltothenumberofpoles oftheopenloop transferfunction.

The accompanying diagram shows where the three poles are. One branch of the root locus on the real axis may be found on the line segment between s=1 and s=0. The line segment towards the left of s=5 represents the other branch of the root locus here on real axis.

Step 2: Using the provided equations, they will determine the values of the centroid as well as the angle of asymptotes. Centroid.

Theangleofasymptotesare,

$\theta = 60^0, 180^0$ and 300^0.

The next graphic displays the centroid and three asymptotes in Figure 10.3.

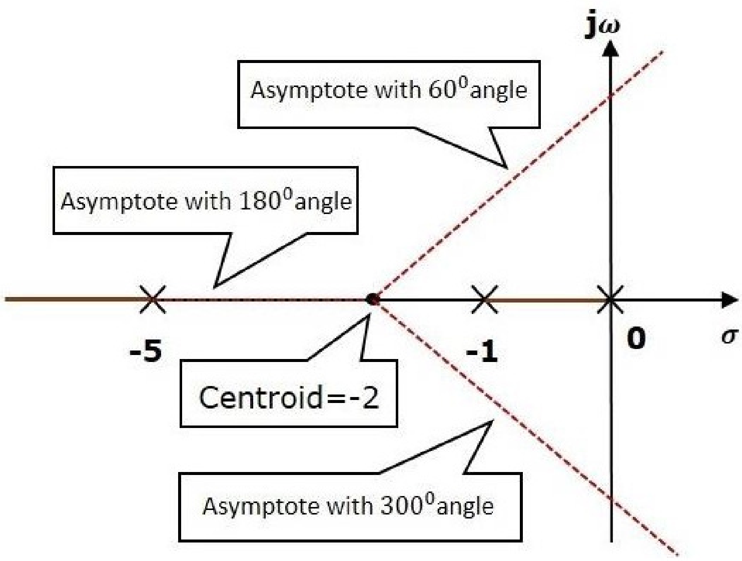

Figure 10.3: Illustrates the displays the centroid and three asymptotes.

Step 3: Two root locus branches cross the hypothetical axis because two asymptotes have angles of 600600 and 30003000. The root locus branches cross the imaginary axis at where there will be one break-away point on the real axis root locus branches between both the poles s =1 and s=0 using the Routh array technique and special case (ii). Following the instructions for calculating the break-away point will result in s = 0.473. Figure 10.4 displays the root locus diagram again for specified control system.

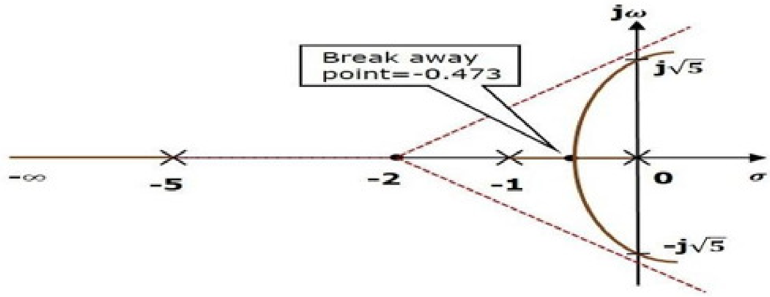

Figure 10.4: Illustrates the root locus diagram again for specified control system.

This will allow you to watch the motion of the closed loop transfer function's poles and generate the root locus diagram for any control system.

We may learn the range of K values for various forms of damping from the root locus diagrams. Root Locus Effects of Adding Open Loop Poles and Zeros, By combining the open loop poles and zeros, the root locus in the's' plane may be moved. Some root locus branches will migrate toward the right side of the "s" plane if a pole is included in the open loop transfer function. The damping ratio decreases as a result. Which indicates that the time domain parameters such as delay time td, rising time tr, and peak time tp decrease as the damped frequency d rises. But it affects the stability of the system. The left side of the "s" plane will be where some of the root locus branches go if we include a zero inside the open loop transfer function. Thus, it will improve the stability of the control system. The damping ratio rises in this situation. Which indicates that the time domain parameters such as delay time td, rising time tr, and peak time tp grow while the damped frequency d decreases. Therefore, we may add (include) the open loop poles or zeros to the transfer function dependent on the demand.

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Questions for Practice

1. How to define concept of control system?
2. How many types of feedback control loop?
3. How many types of feedback system?
4. What are effects of parametric variations on output?
5. What are advantages of open-loop control system?
6. How to measure modelling in the frequency domain?
7. What are electromechanical system transfer function?
8. How are represented time-invariant differential equation?
9. How to calculate number of poles, zeros and transfer function?
10. How to identifying the transfer function?
11. How to identifying feedback relationship?
12. How to calculate the summing points in algebra?